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COMPUTER SCIENCE

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CELLULAR AUTOMATA '86 CONFERENCE

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Foreword

This report contains material distributed at the Cellular Automata '86 conference, held at MIT on June 16-18, 1986. It includes summaries of posters and computer demonstrations given at the conference. The summaries provide a characteristic sampling of current research on the theory and applications of cellular automata.

CA'86 was an open conference, intended to bring together the diverse group of people now working on cellular automata. A cellular automaton is an array (typically one- or two-dimensional) of idendical finite-state computing elements, evolving in time according to a uniform local rule. Cellular automata have been studied from many viewpoints over the past thirty years, and accordingly have known by a variety of names, including tesselation automata, iterative arrays, interacting particle systems, homogeneous computing media, and systolic arrays, among others. Today they are emerging as important models for distributed processes in physics, biology and elsewhere, supplementing the classical approach of partial differential equations. In addition they provide a model for parallel computation which respects at least formally the finite dimensionality of physical space. For an introduction to cellular automata, the reader is referred to D. Farmer, T. Toffoli, and S. Wolfram (eds.), *Cellular Automata,* (North-Holland, Amsterdam, 1984), S. Wolfram(ed.), *Theory and Applications of Cellular Automata,* (World Scientific, Singapore, 1986), and T. Toffoli and M. Margolus *Cellular Automaton Machines*, (MIT Press, to be published).

CA'86 had over 160 participants, drawn from physics, mathematics, computer science, and biology, as well as other fields such as economics and art. It was by many standards an unconventional conference. The primary form of presentation was posters, of which there were about 50. Summaries of these posters comprise the bulk of this report. In addition to the posters, there were many computer demonstrations; at the peak of the meeting, these drew over 60 amperes. Several tutorial talks and discussion sections were also held. Full-length papers describing many of the results presented at CA'86 will appear in forthcoming issues of the new journal *Complex Systems.* The papers will also be published in book form.

Charles H. Bennett
Tommaso Toffoli
Stephen Wolfram

CA'86 organizers

Cellular Automata '86

MIT, June 16-18, 1986

CA '86 was organized by:

Charles H. Bennett (IBM Research, Yorktown Heights)

Tommaso Toffoli (MIT Lab. for Computer Science)

Stephen Wolfram (Center for Complex Systems Research, University of Illinois at Urbana-Champaign)

with administrative support from:

Gayle Fitzgerald (MIT Campus Information Services)
Tania Erlij
David Zaig

The conference was made possible by support from:

IBM
MIT Laboratory for Computer Science
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Thinking Machines Corporation

The following kindly loaned equipment and other items:

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Ridge Computers
Sun Microsystems
Systems Concepts

Note: This conference booklet will be available after the conference as an MIT Laboratory for Computer Science Technical Report.

Cellular Automata '86: Poster Presentations

(as of June 12)

Peter Albin (CUNY): **Economic data structures**

H. Axelrad, B. Giraud and C. Bernard (CHU Pitie Salpetriere): **Realistic dynamics of neural networks models with discrete time iterations: restricted number of simultaneously "active" elements**

S. C. Baer: **Problem solving cellular automata**

B. Boghosian and D. Levermore (Livermore): **A cellular automaton for the Burgers equation**

T. Bohr (Oersted Inst.), G. Grinstein (IBM), Yu He and C. Jayaprakash (Ohio State University): **Coherence, chaos and broken symmetry in classical, many-body dynamical systems**

R. Brower, R. Giles (Boston University) and G. Vichniac (MIT): **Oscillatory solutions in cellular automata Ising models**

David B. Brown (University of Toronto): **Rule competition in cellular automata**

Christopher Burges and Stephane Zaleski (MIT): **Thermal effects in lattice fluids with nearest neighbor interactions**

Michael E. Colvin and Frank H. Eeckman (UC Berkeley): **A cellular automaton model for the cerebral cortex**

T. Cloney, E. Goles and G. Vichniac (MIT): **Iterative integer functions as pseudo cellular automata: the *3x+1* example**

J. P. Crutchfield (UC Berkeley) and N. H. Packard (University of Illinois): **Bifurcations in discretized spatially-extended systems**

K. Culik II (University of Waterloo): **On the power of undirectional and totalistic cellular automata and the existence of a small universal CA**

Jonathan Doner, Stuart Hinds and Jefferson Doner (University of Virginia): **Organizational automata**

Stephen W. Fosdick: **Three dimensional displays of cellular automata**

Peter Gacs (Boston University): **Reliable computation in 3, 2 and 1 dimensions**

David Griffeath (University of Wisconsin): **Stochastic cellular automata**

Gerhard Grossing and Anton Zeilinger (University of Vienna): **Quantum cellular automata**

Puhua Guan and Yu He (Ohio State University): **Bound of short cycles of border decisive automata**

H. Gutowitz, J. Victor and B. Knight (Rockefeller University): **Local structure theory for cellular automata**

H. Hartman, W. Klein, P. Tamayo and G. Vichniac (MIT and Boston University): **Statistical mechanics of inhomogeneous cellular automata**

B. Hasslacher and T. Shimomura (Los Alamos): **Lattice gas hydrodynamics**

M. I. Hofmann (University of Toronto): **A cellular automaton model based on cortical physiology**

Lyman P. Hurd (Princeton University): **Formal language characterizations of cellular automaton limit sets**

Poster presentations

Joel D. Isaacson (Southern Illinois University and IMI Corp.): **Dialectical cellular automata for low-level intelligent computing**

Erica Jen (Los Alamos): **Global properties of elementary cellular automata**

Erica Jen (Los Alamos): **Pattern recognition using cellular automata**

Steven D. Kugelmass and Kenneth Steiglitz (Princeton University): **Architectures for cellular automata machines**

Christopher G. Langton (University of Michigan): **Virtual state machines in cellular automata**

Yann Le Cun (Ecole Superieure): **Learning algorithms for multilayer networks of threshold automata**

Myoung S. Lee and Gideon Frieder (University of Michigan): **Configuration of defective cellular array**

Wentian Li (Columbia University) and Stephen Wolfram (University of Illinois): **Cellular automata spectra**

Daniel G. Maeder (University of Geneva): **The free energy concept in CA models of phase transitions**

N. Margolus (MIT): **Partitioning cellular automata and other topics**

G. McNamara and G. Zanetti (University of Chicago): **Direct measure of viscosity in a lattice gas model**

Bruce Nemnich (Thinking Machines) and Stephen Wolfram (University of Illinois): **Cellular automaton hydrodynamics**

Andre J. Noest (Netherlands Inst. for Brain Research): **New universality for stochastic CA-models of disordered excitable media**

A. Noullez (Universite Libre de Bruxelles), D. d'Humieres, P. Lallemand and Y. Pomeau (Ecole Normale Superieure): **2-d lattice gas cellular automata compressible flows**

N. H. Packard (University of Illinois): **Deterministic lattice models for interface dynamics**

Charles Platt (New School for Social Research): **Simplified cellular-automaton software for the IBM personal computers**

Ken Porter (Systems Concepts), Norm Margolus and Tom Toffoli (MIT): **CAM-6: a cellular automaton machine**

Jerome Rothstein (Ohio State University): **Bus automata: cellular automata with global dynamic variability in choice of effective neighbor sets for each cell**

Jim Salem (Thinking Machines Corp.): **Cellular automata on the Connection Machine: The cellular automaton toolkit**

Rob Shaw (Institute for Advanced Study): **Information density near a phase transition**

Brian Silverman (Logo Computer Systems): **Cellular automata circuits**

Kim Strohbehn et al. (Johns Hopkins University): **Demonstration of a pipelined VLSI based cellular automaton processor**

John Stamey and D. E. Stevenson (Clemson University): **Time-varying cellular automata**

Karl Svozil (Technical University Vienna): **Evidence that quantum fields are no cellular automata**

N. Tufillaro, J. Reilly (Bryn Mawr College) and R. Crandall (Reed College): **Cellular automata as pop art**

Poster presentations

Gerard Y. Vichniac (MIT): **Boolean calculus on cellular automata**

Gerard Y. Vichniac (MIT): **Learning physics with cellular automata**

P. C. Williams (Brown University): **Continuous time cellular automaton simulator**

Stephen Wolfram (Thinking Machines and University of Illinois): **Minimal cellular automaton approximations to continuum systems**

David A. Young (Livermore): **Simulation of biological and social systems with cellular automata**

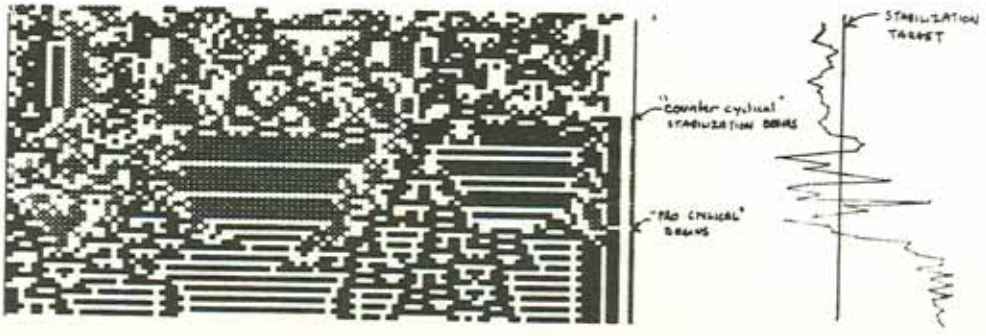
ECONOMIC DATA STRUCTURES -- Peter Albin, CUNY

The familiar characteristic diagrams of the time evolution of 1dCA's are reminiscent of a common format for economic data -- rectangular arrays giving observations on firms (rows) over time. The resemblance is more than superficial. There are several important classes of dynamic economic models in which a firm's expectations and actions are strongly influenced by recent actions of its economic neighbors: competing firms within an industry, suppliers, customers. In many of these models the relevant expectational information is qualitative in nature and representable by state variables: Are firms in the neighborhood now discounting a previously stable price? Do neighboring firms' actions signal a (weak, normal, strong) market?

After appropriately specifying economic "closeness" in the model topology, one can employ the 1dCA as an experimental framework for exploring the statistical properties of realistic economic data and problems faced by intelligent agents who use such data. Appropriate contexts include: a) clustering behavior in spatial markets -- e.g., "price wars" among gasoline stations with overlapping market areas along a stretch of highway, also pollution and crowding effects; b) stability in capital accumulation where the investment decisions of individual firms are interactively based on expectations data; c) persistant inflationary processes, e.g., the diffusion of "petroleum shocks;" d) problems of stabilizing the macro-economy with non selective policy instruments.

The research tools are essentially those used by Wolfram to compile qualitative dynamics and statistical properties. Among matters of interest to economists are : 1) implications to forecasters of computation-irreducible data structures; 2) relationships between firm decision criteria and the qualitative system dynamics they induce; 3) effects on system properties of complex within-industry interactions such as oligopolistic price leadership; 4) qualitative effects when firms "adapt" their expectations; 5) tendencies towards self-organization or "sorting" as occurs in "racial tipping;" 6) qualitative effects on system dynamics of "stabilization" attempts.

The last situation is illustrated below for a 3-state model of capital accumulation specifying a 5-site neighborhood of firms. Grey, white, and black represent below-normal, normal, and above-normal investment, respectively (values: -1, 0, +1). "Normal" investment is that level of firm investment which is consistent with sustainable economy-wide growth [Albin, 1986]. The graph to the left plots the algebraic sum of firm actions. After distributing an initial shock to the system, the model is run for 20 periods to establish its dynamics. After period 21 the Federal Reserve; which is, in effect, a neighbor of each firm; passes a control signal to the system according to whether the sum of site values is above or below an action threshold. The signal -- e.g., a change in interest rates -- is perceived by each firm as the equivalent of an action by a firm in its neighborhood. As can be readily seen, the system trajectory is dramatically altered by the intervention ("countercyclical" in periods 21-40, "procyclical," thereafter). The 1dCA approach provides the first demonstration of a long-suspected effect.



REALISTIC DYNAMICS OF NEURAL NETWORK MODELS WITH DISCRETE TIME ITERATIONS : RESTRICTED NUMBER OF SIMULTANEOUSLY "ACTIVE" ELEMENTS.

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Cellular automata and/or spin glass type mathematical treatements are now commonly used to mimick and gain better understanding of such global problems of CNS functionning as "memory" or "pattern recognition". In particular the Little-Hopfield type models of formal neuron networks have recently allowed much numerical as well as analytical progress. However these approaches tend to underestimate the strenght of all the biological constraints with which the CNS has to cope, as is apparent from morphological as well as physiological data.
In particular, in the original McCulloch-Pitts theory and in von Neumann's cellular automata the temporal coordinate is represented by a sequence of discrete instants, with all changes in the activity of the network happening between one instant and the other. Moreover, integration of Hebb's rule for synaptic changes implies, in these networks, random sequences of excitatory (+) and inhibitory (-) signs assigned to the elements, with essentially as many (+)'s as (-)'s. In real neuronal structures physiological evidence indicates, on the contrary, that at a discrete instant there are many more "silent" neurons than "active" ones.
In order to assess the ratio of active elements to total number of elements we have used a realistic dynamic modelisation of a network of cellular automata.
1- network size was in the range 50 to 500.
2- "activity" was implemented in each neuronal automat as a train of 1msec duration spikes.
3- a random number generator driven by different functions (Gaussian, Poisson,..) gave the instantaneous interspike interval (isi) with only the global mean isi being fixed.
4- the fixed levels of mean isi's were in keeping with physiological data recorded from different structures (40 ; 70 ; 100 msec.).
The calculation of the abovementionned ratio in these conditions shows that, contrary to what is generally implied, only a fraction - in the order of 10% - of the total number of elements are active at a discrete time instant.
It thus appears necessary to develop methods able to integrate such constraints in existing models.

Problem Solving Cellular Automata

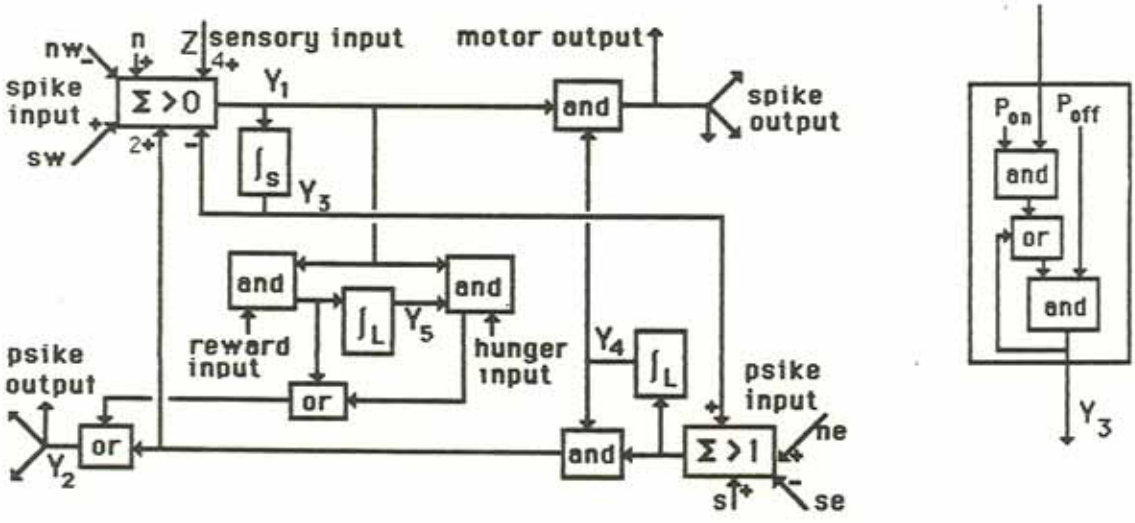
S. C. Baer

A neural machine for association and problem solving is described in the cellular automaton format, and results with its implementation on CAM 5 briefly discussed. In the past, many neural network association machines have been proposed and simulated, but these have not dealt with problem solving. In a typical neural association machine, containing time ordered Hebb synapses (which strengthen when and only when postsynaptic activity *follows* presynaptic activity), the presynaptic cell codes for events generally happening before events coded by the postsynaptic cell, ie., their "causes." Hebb synapses contain a *synapse strengthening signal* which travels backward from postsynaptic cell to synapse, and therefore (in the time ordered case) passes from neural regions representing events to regions representing their causes.

Besides triggering synaptic strengthening, backward signals in the present machine search for goal causes, like the backward chaining signals in Newell, Shaw, and Simon's General Problem Solver. To tap the previously unused problem solving talent of Hebb backward signals, the machine contains l)means synthesizing backward signals in cells representing current goals, 2)means extending the range of the backward signals further than in Hebb synapse models (to allow computed goal causes to search for*their causes* etc.) and 3)means transducing the backward signals into "instant replay" of recent forward signals through a cell (to highlight computed goal causes which are also in the immediate environment). Since spikes carry forward information through the brain, the backward signals are called psikes.

In the present cellular automaton implementation, each cell is characterized by five bits, $Y_1(t), \ldots, Y_5(t)$. Only Y_1, Y_2, and Y_4 directly influence neighboring cells, and correspond to "presynaptic spike output," "psike output," and "output synaptic strength" respectively. Of the 8 nearest neighbors, the n (north) and sw neighbors supply excitatory spike input, nw supplies inhibitory spike input, s and ne supply excitatory psike input, and se supplies inhibitory psike input. Rules for innate and learned connections between cells were chosen to maintain forward-backward reciprocity, so if spikes from cell A reach and excite cell B, in general, psikes from B reach and excite A. Each cell also has a one bit "sensory input" line Z(t), a one bit "hunger input" line, and and a one bit "reward input" line. Y_3 is the output of a rapidly decaying integrating circuit "short term memory" which monitors recent spike input to the cell; its output both gates psike conductivity and feedback regulates average spike output. Y_4 is the output of a slowly decaying integrating circuit "long term memory" which monitors long term psike flow and regulates "synaptic conductivity" for both spikes and psikes. Y_5 is the output of a long term integrator measuring correlation between reward and a cell's presynaptic spike output. The transition rules for each cell are shown by the diagram below. The integrating circuit is shown in detail; P_{on} and P_{off} are random binary variables of probabilities, for example, of 3/16 and 15/16 respectively for the short term integrator, and 1/16 and 255/256 for the long term integrators.

This machine, implemented on the CAM 5 in the MIT Lab. for Computer Science in collaboration with Pablo Tamayo (thanks to Tommaso Toffoli and David Zaig) has already suggested "associative learning." Tests for its problem solving ability are underway.



A Cellular Automaton for the Burgers Equation

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Abstract

We study the approximation of solutions to the Burgers equation

$$\partial_t \rho + c\partial_x(\rho - 1/2\ \rho^2) = \nu \partial_{xx}\rho \qquad (1)$$

by spatially averaging an automaton motivated by random walks on a line. The game consists of moving "particles" on a one-dimensional periodic lattice with speed one and in a random direction subject to the exclusion principle that at most one particle may move in a given direction from a given lattice site, at a given time. The exclusion principle gives rise to the nonlinearity in (1) and introduces correlations between the particles which must be estimated to obtain statistical bounds on the error. These bounds are obtained in two steps. The first is showing that the ensemble average of the game is a stable explicit finite differencing scheme of (1) over the lattice with a second order convergence in the lattice spacing. The numerical diffusion of this scheme plays an important role in relating the automaton rules to (1). The next step is showing that the spatial averaging of a single game converges to the spatial averaging of the ensemble as $M^{-1/2}$ where M is the number of lattice sites averaged. Calculations will be presented.

Coherence, Chaos, and Broken Symmetry in Classical, Many-Body Dynamical Systems

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Abstract

We have studied the effect on the non-stationary states of discrete dynamical many-body systems of the fluctuations neglected in zero-dimensional approximations. Our results have been derived in the context of noisy coupled maps on a d-dimensional lattice, though they apply equally well to fully probablistic cellular automata. Analytical arguments and numerical simulations are employed to deduce the phase diagram in the space of noise amplitude and the control parameter of the map. A noteworthy point is that there is no distinct chaotic phase, i.e., no phase in which the time translation symmetry is completely broken so that the average value varies chaotically in time. The nature of the transitions have also been investigated.

Oscillatory Solutions in Cellular Automata Ising Models

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The special case of nearest neighbor "heat bath" algorithms implemented in serial or parallel (i.e., as cellular automata) are realizations of some of the ideas discussed by Grinstein, Jayaprakash, and He (*Phys. Rev. Lett. 55 (1985) 2527*). In particular, the parallel Heat Bath Monte Carlo dynamics can, by a remapping of variable labels, be shown to be equivalent to a pair of interleaved non-interacting serially updated theories. This example shows that, contrary to the expectations of Grinstein *et al*, there can be Hamiltonian Systems with oscillatory behavior. Some observables (e.g., the staggered magnetization) are not stationary. The oscillatory solutions thus have a non-Boltzmann character, although they are regular Perron-Froebenius eigenvectors of the Markov process. This rich behavior is characteristic of cellular automata; it does not occur in standard statistical mechanical models (cf. Bennett and Grinstein, *Phys. Rev. Lett.* 55 (1985) 567).

Rule Competition in Cellular Automata

by David B. Brown
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A fundamental mechanism in the evolution of biological systems is competition. The competitors are individuals and the fittest survive to dominate the population as a whole. Each organism exhibits a complex range of behavior that is selected as a single coupled behavioral set.

Competition and selection can be drawn into cellular automata theory by introducing several automaton rules and allowing them to compete for cellular space. Competition takes place in regions where different rules are locally adjacent and selection is based upon a master rule.

The pairwise competition between all k=2, r=2 totalistic rule one dimensional cellular automaton rules is examined. The class of master rules, used in the survey, describe rule evolution over a ring solely on the basis of site states and reduce to an identity mapping when only one automata rule is present in a region. Rule choice by the master rule class is limited to those rules found in the neighborhood of a site.

Outcomes of the pairwise competition were , in almost all cases , independent of the specific master rule chosen from this class of master rules.

Bibliography

1. Wolfram , S. Universality and complexity in Cellular Automata, Physica 10D (1984) 1-35

THERMAL EFFECTS IN LATTICE FLUIDS WITH NEAREST NEIGHBOR INTERACTIONS

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There has been much interest recently on the modelling of fluid phenomena using lattice gas models that can be implemented as a cellular automaton. In the models proposed so far, fictitious particles travel on a regular lattice, conserving momentum and particle number when they collide.the velocity of the ficitious particel is always one or zero, so that at each time step, a lattice node interacts only with its nearest neighbors, allowing an easy implementation as a CA. The average properties of the fluid are now the same as those of an incompressible fluid obeying the Navier Stokes equations.

To extend this model to other physical situtions in fluids would be of great interest. An obvious improvement would be to add thermal effects to the models. As they now stand, all particles have velocity one and the temperature is fixed. A simple cure would be to allow for several velocities. This however implies that one allows particles to hop to distant sites. The simplicity of the model with nearest neighbor interactions is lost. We have investigated various ways of introducing thermal effects while allowing particles to jump to nearest neighbors only.

To model some thermal effects, it is important to notice that in many flows of geophysical or technological importance, the thermal effects come in only through a buyant force, and can be neglected altogether otherwise. Thus it is not necessary to use the equation of state of the fluid but to derive the buoyant force. One thus obtains the Boussinesq approximation to the Navier Stokes equations. We do the same approximation for the lattice gas and introduce the buoyant force via an external force field acting on charged particles. The resulting equations are the Navier Stokes equation for the mixture of the two type of charged particles and the diffusion equation for the charge density. This exactly the Boussinesq approximation, and it also applies to salt convection effects.

The preliminary result we present include a derivation of the diffusion equation for the charge, and a numerical check of the analytical expression for the diffusion coefficient. The Chapman Enskog and Boltzman equation approach yield very satisfactory results for this coefficient. This in turn allows to predict the range of Rayleigh numbers realizable with existing computing facilities. Methods for varying arbitrarily the diffusion coefficient and viscosity are presented.

ITERATIVE INTEGER FUNCTIONS AS PSEUDO CELLULAR AUTOMATA: THE 3x+1 EXAMPLE

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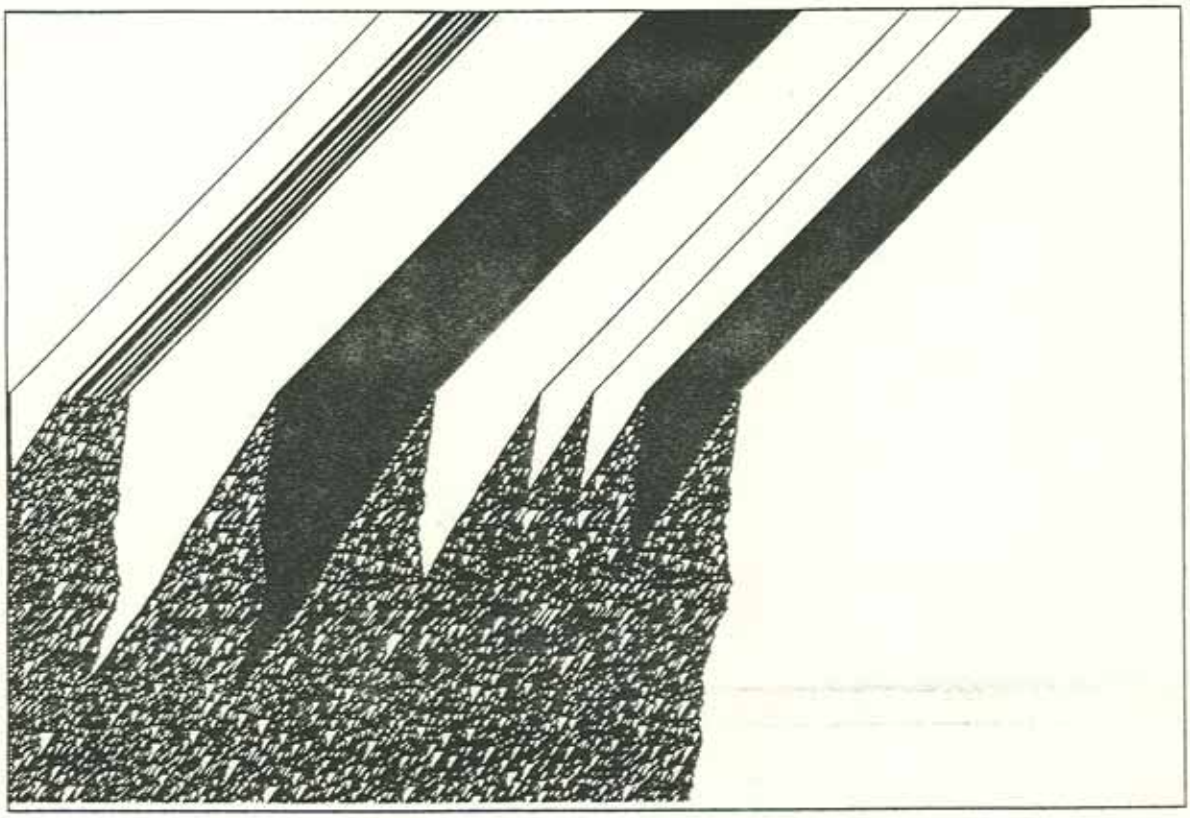
When the iterates of an integer function are tabulated in base 2, they may exibit patterns similar to those of one-dimensional cellular automata. The bit patterns make visible important features of the evolution of these iterates. The figure shows the first 751 iterates of the '$3x+1$' function

$$f(x) = \begin{cases} 3x+1 & \text{if } x \text{ odd} \\ x/2 & \text{if } x \text{ even} \end{cases}$$

with an initial value of

$$2^{350} + (2^{300} - 1)2^{600} + 3^{30}2^{400} + 2^{750} + 2^{850}.$$

The well-known unproven conjecture[2] is that, for all initial conditions, the iterates eventually enter a 4-2-1 cycle, i.e., the bit pattern shrinks to the left.



[1]Current address: Dept. Math. Esc. Ing., U. Chile, Casilla 170, Correo 3, Santiago, Chile.

[2]See the recent reviews by B. Hayes (*Sci. Am.* 250:1 (Jan. 1984) 10–13), and by J. C. Lagarias (*Am. Math. Monthly* 92 (Jan. 1985) 3–22).

A Cellular Automata Model for the Cerebral Cortex

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We have constructed a general mathematical model for cortical tissue. This model is based on a three state stochastic cellular automata with a hexagonal close packed array of cells. Each cell can represent either a single neuron or a highly connected group of neurons.

We have implemented this model in a computer program capable of using any specified interaction rule to update synchronously or asynchronously a lattice of up to several hundred thousand cells. The program incorporates a Hebb-type modification routine to allow continuous modification of synaptic strengths. We have used this model to investigate the emergent properties of a network of cells interacting via an "inverse Laplacian" function. More precisely, each cell is connected to its six nearest neighbors by an inhibitory synapse and to its twelve next-nearest neighbors by an excitatory synapse. Such an interaction rule has been suggested by the anatomical work of Szentagothai[1] and the theoretical work of Palm[2].

This system displays content addressable memory. Furthermore, this system can support long lived, regular oscillations whose period and amplitiude are determined by the initial activity patterns. The system can tolerate a certain degree of asynchronicity in updating without qualitative changes in the oscillations or the system's capability to act as a content addressable memory.

[1] J. Szentagothia, Proc. R. Soc. Lond. 201 219, 1978.

[2] G. Palm, Neural Assemblies (Berlin: Springer 1982).

Bifurcations in Discretized Spatially-Extended Systems

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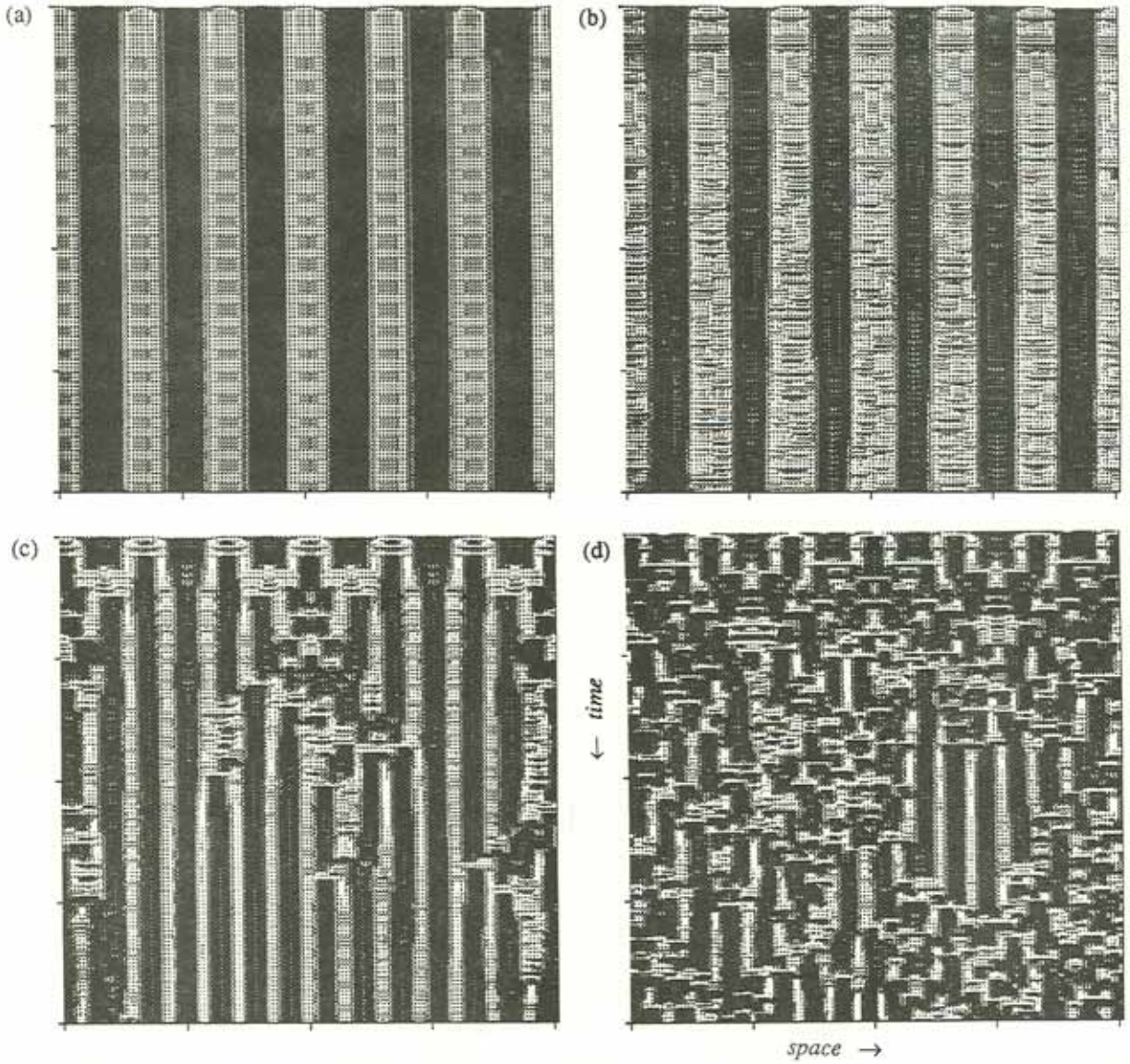

We examine the qualitative behavior of spatially-extended systems discretized in time and space with continuous local variables, called *lattice dynamical systems*. When these systems are approximated using a finite number of states at each site, the resulting system is a cellular automaton. The number of states per site, or resolution, can be thought of as the arithmetic precision with which a continuous-state lattice dynamical system is simulated. As parameters are varied, a continuous family of lattice dynamical systems is obtained. Qualitatively different behavior along this path is separated by structurally unstable systems at bifurcation points. The discrete sequence of cellular automaton rules obtained by discretization inherits the bifurcation structure of the one parameter family of lattice dynamical systems. In fact, we find that bifurcation sequences are preserved even under radical discretization.

A typical bifurcation sequence is illustrated in the figure above. The dynamical rule giving the time evolution of each site value a_i is a composition of two simple functions: first the discrete approximation of the diffusion operator is implemented by taking the average of a site with its neighbors, $\hat{a}_i = (a_{i-1}+a_i+a_{i+1})/3$, and then this value is operated on by a simple one dimensional map, $a'_i = f_\lambda(\hat{a}_i) = \lambda\hat{a}_i(1-\hat{a}_i)$. The sequence illustrated is obtained by varying lambda: *(a)* λ=3.5, locally periodic behavior; *(b)* λ=3.7, local chaos; *(c)* λ=3.8, the transition to global chaos; *(d)* λ=3.9 global chaos.

The bifurcation to global chaos *(c)* displays complex spacetime transients; in a cellular automaton finite-precision approximation, these become propogating structures similar to those found in cellular automata proposed to be computation universal. Thus, we find that space-time structures of potential use in cellular automaton engineering are found in systems at bifurcation points.

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On the Power of Undirectional and Totalistic Cellular Automata and the Existence of a Small Universal CA

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S. Wolfram calls a cellular automaton totalistic if the states of its cells are integers and the next state of each cell is a function of the sum of the states in its neighbourhood. D. Gordon [G] has shown that every Turing machine can be simulated by a totalistic linear cellular automaton. It is also easy to show that every CA can be simulated by a totalistic one (which implies the former). We generalize the latter result to every (finite or infinite) "regular" systolic network, with a very broad meaning of regularity.

A unidirectional (one-way) CA is a linear cellular automaton in which the neighbourhood for each cell is ■□. As a second "normal form" result we show that each CA can be simulated by twice slower unidirectional one. This is perhaps surprising, but quite easy to show using the "topological transformations of unrollings" (see [CF], [CY]). Finally, we strengthen the result of Smith [S], that there exists a computationally universal CA with 18 states, i.e. a CA which given a description of a Turing machine M and input x simulates the computation of M on x. We give a universal CA with 16 states which can simulate every other CA with any initial configuration (see problem 15 in [W]). Moreover, if we are willing to increase the number of states we can use the above normal form results and make our universal CA totalistic and unidirectional. However, the construction of our universal CA uses the above normal forms even if we do not want to make it unidirectional and totalistic.

References

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Organizational Automata

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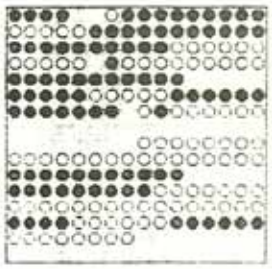
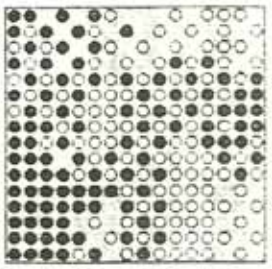
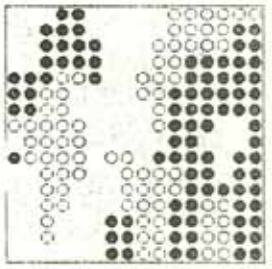
Cellular automata (CA) are arrays of elements which compute global functions on the basis of local transition rules. In most recent work on CA (e.g., see 1), automata transitions are governed by deterministic counting rules over the Moore or Von Neumann neighborhoods. We present results concerning a different class of CA that are stochastic functions of variably structured neigborhoods. Our interest in this class of CA derives from our modelling of reorganization processes in neural memory driven by random decay.



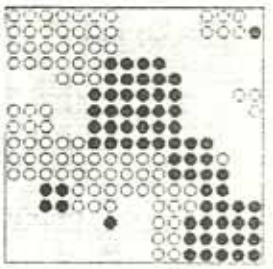
Consider an NxM array, Z, of cellular automata. Each CA, can take on any of three possible states. State transitions in Z are determined by a stochastic process such that automata randomly change state over time (random decay). The transition of any given CA, however, is inhibited as a function of a decay parameter and the quantitative organization of the local neighborhood of the cell. In other words, generally, the more organized the neighborhood of an automaton the lower the probability that it will undergoe state transition.



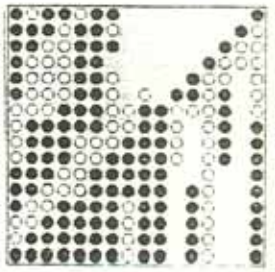
Each CA computes organization for one or more neighborhoods consisting of some configuration of four adjacent cells. Organization is determined on the basis of both local and global properties of Z. A detailed explanation of the organizational function is given in (2). We have examined a number of characterizations of the transition function. In the results discussed below, state transitions are modulated relative to an organizational threshold. The probability of transition goes to zero as the local organization approaches the threshold value.



Results indicate that random initial patterns can self-organize as a function of the structure of the local neighborhood and the threshhold value. Examples of organized patterns for varying neighborhoods are shown in the accompanying figures. Succesive incrementing of the threshold increases organization up to a critical limit. Beyond this limit organization rapidly destablizes and undergoes random fluctuation. Self-organization involves two stages. In the first, organization is triggered by a select class of subpatterns in the array. The second stage is marked by the segregation of subpatterns into high and low frequency components. A further result is that competition among neighborhoods of different configurations can modify organization.



We are extending this approach to processes involving the interaction of multiple arrays of CA. This work has application to theories of memory and the understanding of organizational complexity. In addition, it may aid in the development of fault-tolerant memory systems.



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THREE DIMENSIONAL DISPLAYS of CELLULAR AUTOMATA

by
STEPHEN W. FOSDICK

The primary methods for displaying cellular-automation models have been computer line printers and video terminals. This has enabled studies to be made in two dimensions, typically line by line, on a printer or a screen at a time on a terminal.

The motivation to view the output from calculations producing data in a three dimensional spatial domain necessated the need to construct a relatively low cost display capable of producing real images in three dimensions.

The costs for traditional, real-time, three dimensional displays have been beyond the means of many researchers. Real-time display is only important when the output needs to be viewed by a researcher at a rate within human tachistoscopic bandwidth. In many cases,the final display is the end result of many high speed calculations. If the only compromise is to the real-time parameters of the display, then the following method described will produce high resolution, three dimensional cellular automata at low cost.

The method consists of having relative motion between a camera and a computer video terminal. In the interest of moving the smallest mass, the camera was selected to be the moving element. The camera is placed on a linear slide, driven by a stepping motor, controlled from a computer that moves the camera closer or away from the computer screen. The video terminal produces an image one screen at a time in "X" and "Y" .The camera moves in the "Z" direction. By photographing sequential screens, a three dimensional image is created as one would view slices of bread in a semi-transparent loaf. Cell density is adjusted for better viewing of internal areas. Stereoscopic viewing of cellular automata is possible by substituting a stereo camera for a single lens camera.

The above technique can be utilized in an additional manner by substituting the video screen with two stepping motors that can move a light source in concentric spheres. Motor "A"is mounted at a right angle on the shaft of motor "B". The light source is mounted on the motor "A" shaft and has provision for adjusting the radius from the "A" motor shaft.

The conclusion of the work demonstrate that three dimensional analysis of cellular automatons can be performed at relatively low cost, or at lower cost than previous methods, thereby expanding the field of investigation to more researchers.

Reliable computation in 3, 2 and 1 dimensions

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Cellular arrays give, for several reasons, the appropriate formalism for building arbitrarily large computing devices from unreliable elementary components. A related problem is whether, in a large homogenous probabilistic system with no prohibited transitions, structures of arbitrary sophistication can arise.

1. The first known (infinite) cellular array *(medium)* capable to remember a bit of information reliably was constructed in [T 74]. It was conjectured by statistical physicists that no such one-dimensional medium exists. In [G 85], John Reif and the author applied Toom's construction in [T 78] for the three-dimensional real-time reliable simulation of an arbitrary one-dimensional medium. Just as Toom's rule, the three-dimensional medium is simple. Each bit of information is encoded in a two-dimensional array (torus) whose size is logarithmic in the size of the computation (space · time). Within each torus, Toom's rule is performed. The proof method used (the so-called k-sparsity technique) seems to seems to be a powerful one, applicable to different kinds of stochastic homogenous media.

2. A three-dimensional error-correcting medium is not realizable physically, since each component produces heat at a constant rate. A *two-dimensional medium* is possible, sacrificing some simplicity. To initialize the work of this medium, cells must be organized into a hierarchy of *blocks*. Each block has a *program* (the same for all blocks) that the individual cells (subblocks) consult during a *working period* of error-correction.

Three kinds of error can be distinguished (relative to a particular level) in such organizations: information errors, structure errors and program errors. A program or structure error on some level of the hierarchy, will be simply seen as a group of information errors from the viewpoint of lower levels.

Information errors are the ones correctable by traditional encoding-decoding and repetition techniques.

Structure errors affect the parameters of the error-correcting mechanism. This structure can be kept simple and homogenous on each level, so that Toom's rule (applied on each level) suffices for its correction.

A *program error* occurs if the program of a block is damaged. A procedure can restore it, using Toom's rule, from the neighbor blocks, but--a subtlety—this procedure must be administered by the program of the individual cells (subblocks) in the block. As an alternative, the cells may simply *know* the program of the block they are in. This seems to happen in complex biological systems: each cell contains the genetic code of the whole organism.

3. It is possible to build a one-dimensional error-correcting medium, refuting the conjecture mentioned in 1., as is done in [G 83], using some ideas from the "philosophical" paper [K 78]. Structure correction must be done in a more complicated way since there is nothing like Toom's rule in one dimension.

The space redundancy in 2. and 3. can be reduced to a constant factor. However, the simulation is no more real-time: a logarithmic delay is introduced, since repetition is used for the correction of information errors.

Stochastic Cellular Automata

David Griffeath (Mathematics Department, University of Wisconsin)

The mathematical theory of stochastic cellular automata, known to professional probabilists as interacting particle systems, has been a subject of vigourous research over the past 20 years. The recent book by T.M. Liggett, Interacting Particle Systems (Springer, 1985) provides an excellent overview of the subject. Almost all of this theory deals with systems which have two states per site (on/off, infected/healthy, pro/con, occupied/vacant or whatever). I am currently initiating a major project which investigates systems with more than two possible states (*colors*, say) per site, perhaps a very large finite number, or even infinitely many. Such models are usually less tractable than the 2 state systems which are bad enough to begin with. But one can always simulate, hope to find interesting new types of ergodic behavior, and perhaps discover some mathematically tractable results.

For simplicity I am looking at a small but very rich class of dynamics for models in the plane. Namely, each site waits a random amount of time and then "eats" a neighboring site (i.e. replaces the color at that site with its own color). The waiting times are determined by an appetite rule. Models of this variety display varied and interesting behaviors depending on the appetite rule. As a result one gets some intriguing complex computer graphics which explore the interface between order and chaos from the chaotic side. My demonstration will discuss three particularly colorful examples:

1) the classical stepping stone model of population genetics, which exhibits exchangeable clustering among 32,000 colors, say;

2) a cyclic model which seems to exhibit a phase transition in the number of colors -- for small numbers of colors the model clusters, but for large numbers of colors it gets stuck in a final (random) state;

3) a homogeneous smoothing process which clusters in such a way as to form a smooth field of color transitions.

All of these examples are unbiased -- no color has an advantage over any other. For biased systems one can ask which color takes over the world. The results are sometimes paradoxical.

QUANTUM CELLULAR AUTOMATA

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It is evident, that in order to become faster, cellular automaton machines have to get increasingly smaller in size. Therefore a regime will be entered where quantum effects cannot be neglected and ultimately these quantum effects may very well be dominant. Consequently one will not be able anymore to know for certainty whether the value at a given site is 0 or 1 at a given instant of time. Quantum mechanically this fact is described by introducing probability amplitudes. We report results obtained by studying the evolution of one-dimensional cellular automata governed by quantum mechanical rules in such a way that superposition of probability amplitudes is permitted.

The present investigations focus on strictly local (i.e. nearest neighbor) interaction and on unitary evolution. We will present the results in the form of probability maps. These clearly exhibit typical quantum features like constructive and destructive interference, beats and the like. These maps will be classified according to the arising patterns.

Bound of Short Cycles of Border Decisive Automata and Exact Results for Automata with Additive Rules

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A cellular automata rule is called border decisive if the rule of the automata is linear in the variable corresponding to the farthest neighbor. If the number of the neighbors is r, the number of possible states at each site is q, then the number of the stable states of any border decisive automata is bounded by q^{r-1}. The number of cycles with length c is bounded by $q^{c(r-1)}$. Among q^{q^r} automata with r neighbors, there are $2(q-1)q^{q^{r-1}} - (q-1)^2 q^{q^{r-2}}$ automata that are border decisive.

A cellular automata rule is additive if the rule is linear in each variable. Complete state transition diagrams for finite additive CA of any order and in any dimension can be computed in a time polynomial in the number of sites. The state transition diagrams consist entirely of identical trees rooted on cycles. Conditions on the rules which make them reversible are obtained. General formulae for cycle lengths and multiplicities are given.

Local Structure Theory for Cellular Automata, Gutowitz, H., Victor, J., Knight, B. Rockefeller U., 1230 York Ave., Box 179, N.Y., N.Y., 10021-6399

The local structure theory is a generalization of the mean-field theory for cellular automata(Wolfram 1983, Schulman and Seiden 1978). The mean-field theory is a model of cellular evolution which makes the assumption that iterative application of the rule does not introduce correlations between the states of cells in different positions. This assumption allows the derivation of a simple formula for the limit density of each possible state of a cell. The most striking feature of cellular automata is that they may well generate correlations between the states of cells as they evolve. The local structure theory takes the generation of correlation explicitly into account. It thus has the potential to describe statistical characteristics in detail.

The basic assumption of the local structure theory is that though correlation may be generated by cellular automaton evolution, this correlation decays with distance. This assumption allows the derivation of formulas for the estimation of the probability of large blocks of states in terms of the probability of smaller blocks of states. Given the probabilities of blocks of size n, probabilities may be assigned to blocks of arbitrary size such that these probability assignments satisfy the Kolmogorov consistency conditions(Denker 1976) and hence may be used to define a measure on the set of all possible (infinite) configurations. We call the class of measures defined in this way finite (or n-) block measures. An arbitrary measure may be approximated to any degree of accuracy by a finite block measure. A function called the scramble operator of order n maps a measure to an approximating n-block measure.

The action of a cellular automaton on configurations induces an action on measures on the set of all configurations(Lind 1984). We combine the scramble operator with the cellular automaton map on measures to form the local structure operator. The local structure operator of order n maps the set of n-block measures into itself. We hypothesize that the local structure operator applied to n-block measures approximates the rule itself on general measures, and does so increasingly well as n increases. The fundamental advantage of the local structure operator is that its action is explicitly computable from a finite system of rational recursion equations.

Empirical study of a number of cellular automaton rules demonstrates the potential of the local structure theory to describe the statistical features of cellular automata. The behavior of some particularly simple rules is derived analytically. These rules serve as useful checks for the empirical methods employed. Other rules have more complex behavior. Even for these rules, the local structure theory yields an accurate portrait of both small and large time statistics. We undertake a classification study of cellular automata using the local structure theory. Rules which differ in construction may nonetheless lead to the same local structure approximation. It is argued that this provides a useful classification scheme for cellular automata.

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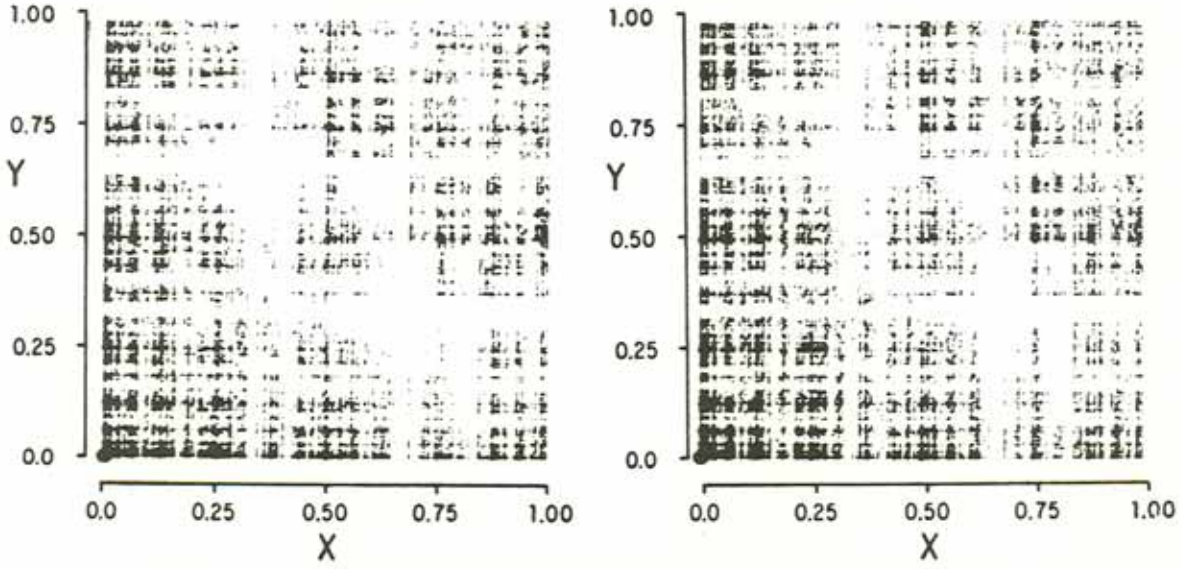
Figure: The invariant measure of r=1,k=2 rule 22. 16-blocks are mapped to the unit square following Grassberger (1984). The probability of each block is proportional to the size of the symbol placed at the block coordinates. Left: Monte Carlo sampling; Right: order 11 theory.



STATISTICAL MECHANICS OF INHOMOGENEOUS CELLULAR AUTOMATA

H. Hartman[1], W. Klein[2], P. Tamayo[3,4], G. Vichniac[3].

We consider an inhomogeneous cellular automaton which consists of two planes, one of which (the back plane) contains the transition rules or programs and the other (the front plane) the evolving data. The data consists of zeroes and ones and the program consists of the XOR and AND rules distributed in a manner described below.

Several cases are considered in both one and two dimensions. The rules can be distributed at random or with correlation and can be either quenched or annealed. By quenched we mean that the rules on the back plane are held fixed during the evolution of the data on the front plane; the dynamics is deterministic, the randomness being inherited from the initial conditions. By annealed we mean that the rules on the back plane are updated as the same rate as the data on the front plane. Clearly interpolations between these two extremes are also possible.

One method for distributing the rules with correlation is to assign the 4-input XOR rule to a down spin and the 4-input AND rule to an up spin in an Ising model which is allowed to evolve with either Glauber or Kawasaki dynamics. The Ising Hamiltonian $-\beta H = \beta \sum_{ij} J_{ij} s_i s_j$ controls the distribution of the rules on the back plane. Here the s_i are zero or one and β is the inverse temperature. The interactions between the rules (spins) are arbitrary and can be taken to be, e.g., ferromagnetic ($J_{ij} > 0$ for all i and j) anti-ferromagnetic ($J_{ij} < 0$ for all i and j) or a mixture such as a spin-glass (the sign of J_{ij} chosen at random). In one dimension we use the phenomenological classification of Wolfram. We observe that:

a) the quenched case gives periodic behavior (class 2);

b) the annealed case gives a mapping into phase transitions (see E. Domany and W. Kinzel, *Phys. Rev. Lett.* **53** (1984) 311-314);

c) a characterization of localized complex behavior (class 4) as a critical slowing down at the second order phase transition was conjectured (stochastic version of the halting problem).

In two dimensions the quenched case is much more complicated due to feedback. We get XOR dynamics on a percolation cluster. We observe a "necking" phenomena due to destructive interference. With Glauber dynamics on the back plane and a single 1 on the front plane we get a spread of the ones on the front plane as the clusters of XOR interact. This can be studied as a function of temperature and its behavior at the critical point monitored. The spread of ones on the front plane is at the "speed of light." In the spin-glass case the spread is slowed down.

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A CELLULAR AUTOMATON MODEL BASED ON CORTICAL PHYSIOLOGY

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Despite the complexity of the brain, there is a certain regularity in the structure. The primary sensory areas are organized into "hypercolumns" [1]. Most of the connections of a neuron are with neurons in the same or a neighbouring "hypercolumn". Given this kind of structure we may attempt to model some aspects of cortical function using a cellular automaton type of model.

In the basic model each cell corresponds to a small local group ("module") of neurons. The procedure for calculating the next generation is based on cortical function. First, the weighted sum of a given cell and its three nearest neighbours is taken. A thresholded stimulus-response (input-output) function is then applied to this sum to obtain the next state of the cell. The weights used may be positive or negative, but only sums that meet or excede threshold (a positive integer) will produce a non-zero next state. This combined with the fact that the weights used are symmetrical means that only "legal" [2] cellular automata will result.

Two basic types of stimulus-response functions have been studied. The first is based on the logistic curve, this in general produces stable or at most period 2 oscillatory behaviour, depending on the weights used. The second is based on a Gaussian curve, this can produce more complicated oscillatory, chaotic or apparently Wolfram's [2] Class IV types of behaviour. The transition from Class III (chaotic) to Class IV behaviour is generally a function of the threshold value used in the stimulus-response function. Increasing threshold leads from chaotic to Class IV behaviour and eventually to universal death.

There do not appear to be any qualitative differences between behaviour in 1 or 2 dimensions. Extensions to the basic model include use of constant (time invariant) forcing function, corresponding to input from a different level of the nervous system and asynchronous calculation of the next generation. These extensions provide the basis for a more realistic model of cortical function.

The parameters used to define a rule in this model have close physiological correlates. The weights correspond to the strengths of connection between two "modules". The threshold is the threshold for excitation of a "module". The stimulus-response function represents the input-output relation of a "module" and depends on the internal organization of a "module". A wide range of behaviour is observed in this model. In general the weights affect the spatial structure of the array of cells. These are presumably relatively constant in the short term, but may be altered by learning. Transitions from one type of behaviour to another are generally effected by varying the stimulus-response function or to a lesser extent the threshold. In terms of physiology these may correspond to dynamically variable parameters of the nervous system that may be related to arousal states (asleep, awake, attentive, relaxed etc.) or pathological conditions (e.g. epilepsy, hallucinations).

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Formal Language Characterizations of Cellular Automaton Limit Sets

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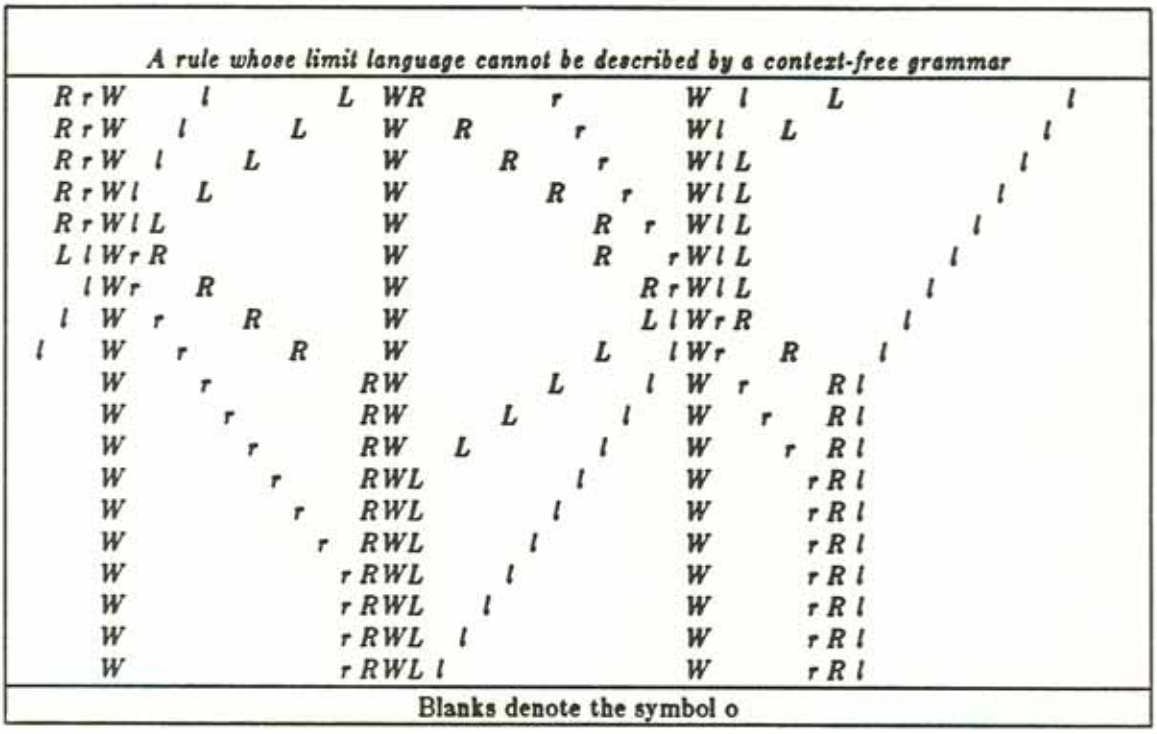
Infinite strings generated by one dimensional cellular automata, are determined by their finite substrings. This correspondence allows the application of formal language theory to sets of configurations. Wolfram has shown[*] that the language reached by a cellular automaton in finite time can be described by a regular grammar. This presentation extends his result by defining analogs of more complicated grammars for CA configuration sets, and illustrating CA whose limit languages exhibit increasing levels of complexity. The final example gives a cellular automaton whose limit set corresponds to a non-recursive language.



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Dialectical Cellular Automata for Low-Level Intelligent Computing

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Summary

To be "dialectical", cellular automata have to be constrained by two conditions: (1) rules are "differential", and (2) boundary conditions are fixed, with a distinct "background" state. In differential rules a new state at a given site is dependent only on the difference/sameness of the site's present state w.r.t. each of its neighbors' states. Natural sensory or perceptual elements (i.e., the retina) are known to operate under differential rules, which motivates the present interest in this particular class of CA, as opposed to the more common CA having totalistic rules [1] or other types of algebraic or logical rules. Those two conditions typically drive a CA into classical dialectical behavior, yielding certain types of novel intelligent computing. A 1-D 4-state nearest-neighbor dialectical CA (k=4, r=1) is discussed and its unexpected behavior described [2]. The instant CA is shown to be subsumed under Post's monogenic normal systems, and also to be a close relative of Wolfram's Rule 126 CA [1]. That bridge appears to give a new perspective on "universality" issues, at least for this class of CA. Features of low-level intelligence emerging from a massive network of such dialectical CA are discussed. Extensions to 2-D CA yield applications relating to image processing and visual perception, and some graphics from image processing results are presented (see example below).

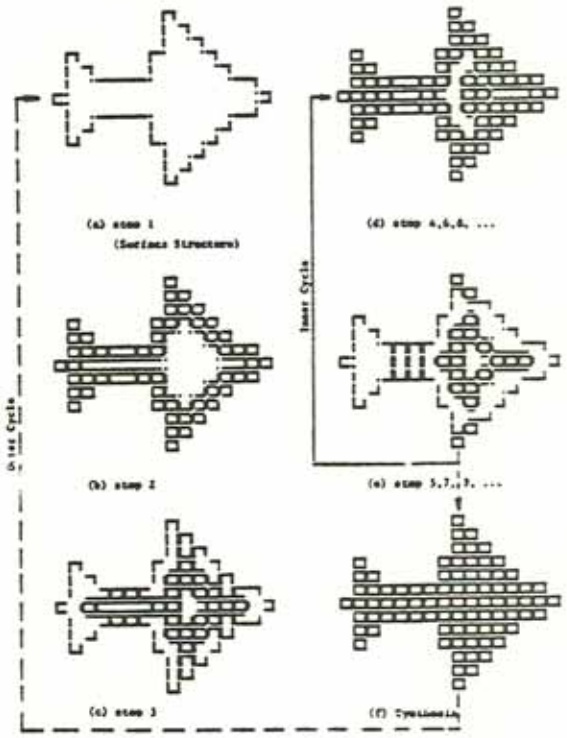

Fig. 1 - Dialectical image processing of a plane silhouette

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GLOBAL PROPERTIES OF ELEMENTARY CELLULAR AUTOMATA

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"Elementary" cellular automata are defined to be one-dimensional automata with binary site values depending only on nearest-neighbor interactions. The diversity of behavior generated by such rules has been widely noted, and a phenomenological classification scheme proposed by Wolfram [1].

An approach has been developed [2] that relies on analysis of the local structure of elementary rules to provide rigorous results characterizing their global behavior. For instance, conditions have been obtained defining the classes of elementary rules for which arbitrary finite initial conditions (i) evolve to a homogeneous state; (ii) generate at least one constant temporal sequence; (iii) generate infinitely many aperiodic temporal sequences. The result on the aperiodicity of temporal sequences generated by certain cellular automata supports a conjecture by Wolfram [3] that these systems may serve as efficient pseudo-random number generators.

A major component of the approach used to obtain the above results is the study of the distinct "deterministic structures" exhibited by cellular automata rules. It is certainly explicit in all elementary rules that the value of a site is determined by the values of its nearest neighbors at the previous time step. It is possible, however, that specific rules may be defined in such a way as to induce a determinism operating along a path other than the time-increasing one. For example, for a certain class of elementary automata, it can be shown that that the values of any two adjacent temporal sequences determine the value of the temporal sequence to their right. Rules belonging to this class can thus be viewed as possessing a deterministic structure in addition to that shared by all cellular automata systems.

The existence of additional deterministic structures for an automaton rule can be shown to be a direct consequence of the many-to-one, versus one-to-one, nature of the rule restricted to certain well-defined subsets. The results describing the global behavior of automata rules are then consistent with intuitive notions of differences between one-to-one and many-to-one mappings. Specifically, it has been shown that both evolution to a homogeneous state and generation of constant temporal sequences require that the underlying automaton rule be two-to-one in a particular sense. On the other hand, rules that are one-to-one in that same sense generate aperiodic behavior.

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PATTERN RECOGNITION USING CELLULAR AUTOMATA

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Certain cellular automata can be shown to possess "invariant strings" (analogous to fixed points). Mathematical results characterizing these invariant strings have been obtained, and their applications to a problem in pattern recognition explored.

An invariant string of a one-dimensional cellular automaton rule is defined to be a finite spatial sequence of site values that remains invariant under the rule, independent of the sequence's spatial position or the values of its neighboring sites. The rules for which invariant strings exist have been characterized [1], together with the set of invariant strings associated with each such rule. Furthermore, given an arbitrary string P, a simple procedure can be used to construct the automata rules under which P is invariant. In particular, it has been shown that a rule of minimum neighborhood size can be defined for which P is the unique invariant string. This rule has the property that arbitrary initial conditions evolve to spatial sequences consisting essentially of concatenations of P. Thus, spatial sequences of this form constitute the only attractors of such automata.

The results on invariant strings have direct implications for the use of cellular automata for pattern recognition. Suppose the problem is to choose a cellular automaton that will "recognize" a particular pattern in an arbitrary input sequence; i.e., to choose an automaton under whose temporal evolution, the desired string is "preserved," and all others "annihilated." The advantage of a cellular automaton approach in this context is that the processing is performed in parallel, and thus could be significantly more efficient than conventional serial searches for problems involving large amounts of input data. The rule of minimum neighborhood size for which a string P is the unique invariant string is then the optimally efficient rule that possesses the desired quality of preserving P and annihilating all other patterns. Moreover, the number of steps required for this rule to perform the recognition task is bounded above by a quantity depending only on P, and thus is independent of the length of the input data.

The analysis described above has been extended [2] to provide algorithms based on cellular automata for the recognition of multiple distinct and/or "noisy" patterns. Applications include the recognition of "crossovers" (i.e., exchanges) between two sequences, and the simulation of "hetero-associative" memory requiring the mapping of specific patterns A_i to other patterns B_i.

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Architectures for Cellular Automata Machines

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There has been a recent flurry of activity in applications of cellular automata (CA). They appear to provide an alternate approach to the solution of certain classes of problems including, but not limited to, fluid turbulence, lattice spin systems, lattice quantum field theories, n-body problems, and number theory. [1]

The hope is that these new cellular-automata based techniques will make it possible to build more highly parallel hardware, and thereby surpass the performance of conventional numerical methods, even on parallel arithmetic-based machines.

We need machines capable of performing many site updates per second on huge lattices of cells, typically on the order of 10^9 Updates of Lattice Points per Second (GULPS). Supercomputers like the Cray XMP-48 can provide on the order of 1 GULP, but at a cost of many millions of dollars. The Thinking Machines Incorporated's Connection Machine provides similar performance at a lower cost because of its greater parallelism, but still costs more than a million dollars. Margolus, et. al., [2] have proposed special purpose machines to perform high speed CA updates but the cost of such machines may also be high.

It appears that when the algorithm is very simple and likely to be fixed, a custom VLSI implementation can provide a good price/performance ratio, and give the individual user a useful machine at nominal cost. The major problem is the inevitable difficulty of matching memory and processor bandwidths, and making effective use of silicon area.

We will describe three systolic system architectures for the two-dimensional problem: *series*, *parallel*, and *series/parallel*. We then analyze the choice of critical parameters, and derive the relationships between throughput rate and area for each choice.

We also describe preliminary designs for series and series/parallel prototype boards for a SUN-3.

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1. Stephen Wolfram, "Scientific Computation With the Connection Machine," Unpublished Manuscript (March 1985).
2. Norman Margolus, Tommaso Toffoli, and Gerárd Vichniac, "Cellular Automata Supercomputers for Fluid Dynamics Modeling," Technical Memo LCS-TM-296, MIT Laboratory For Computer Science, Cambridge, MA (December 1985).

Virtual State Machines in Cellular Automata

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One of the most interesting properties of cellular automata is their capacity to support propagating structures, such as the familiar "glider" of Conway's game of *life*. These propagating structures can be viewed as *virtual automata*: automata that are *embedded in* the very "tape" upon which they compute and that are *constituted of* the very symbols which they use to "write" on the tape in the course of their computation. These virtual automata, or *virtual state machines* (VSM's), can be viewed as either processes or data, and can create, modify, or erase other such VSM's. Thus, VSM's can support a dynamic, distributed logic based on interactions between semi-autonomous, free-ranging automata; a logic whose *operators* can also be *operands*.

Propagating structures in cellular automata are the discrete analogues of solitary waves or *solitons* in physical systems. They also behave much in the manner of *enzymes*, the active agents in biological cells. When viewed as systems of interacting automata, one may begin to separate the *logic* of such naturally occurring phenomena from the details of their particular physical implementation, bringing them within the purview of formal automata theory. In turn, such natural phenomena can serve as the basis for new models of massively parallel computation. Thus, VSM's seem to be particularly important structures, for both theoretical and practical reasons.

It turns out that systems dominated by interacting VSM's arise naturally in certain classes of cellular automata. In particular, as one varies a parameter controlling the "reactivity" of the individual cells over a certain range, one observes the emergence of more and more complex systems of interacting VSM's. These systems show the full spectrum of fixed-point, periodic, and chaotic behavior known from the study of dynamical systems. The most interesting of these "naturally occurring" systems of VSM's seem to be found at the onset of chaotic behavior.

This presentation will include demonstrations of interacting systems of VSM's using a cellular automaton simulator running on an APOLLO DN660 color workstation. These demonstrations will include some which should be of interest to physicists and others which should be of interest to biologists. For example, I will demonstrate systems that exhibit dendritic growth, systems that reproduce themselves, and systems that simulate the dynamics of an insect colony.

LEARNING ALGORITHMS FOR MULTILAYER NETWORKS OF THRESHOLD AUTOMATA

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We present some theoretical and experimental works on a learning procedure for multilayer threshold networks, recently proposed by several authors independently [1][2]. This procedure has been originally designed for multilayer feed-forward threshold networks, but is easily extensible to cross-connected networks.
Threshold networks are made of interconnected threshold automata (TA) which compute a weighted sum of their inputs, and produce a +1 output value if the sum exceeds zero and -1 otherwise. Several algorithms are known that find the weights of a single TA, given examples of desired input-output pairs. however, since the percentage of boolean functions in n variables that are computable by a TA tends rapidly toward zero as n increases, most learning TA networks have a very limited computational power. Extensive studies have shown the consequences of this limitation when the network has only one layer of automata with modifiable weights [3]. This limitation applies also to most cross-connected TA networks, including one proposed by Hopfield [4] for modeling associative memorization.
Multilayer feed-forward networks do not have this limitation. They are composed of several groups of TA, the first group is the input layer whose state is clamped externally, it is followed by one or several layers of 'hidden' TA. The last group is the output layer. Connections between TA are allowed only from lower layers to higher layers. When an input vector is given to the network, the output is computed by propagating through the successive layers. The learning procedure iteratively computes a set of weights which minimizes the average error between the actual output of the network and the desired output given by the teacher for every input pattern. This procedure uses a back-propagation process that computes a 'desired state' associated to each hidden automaton, It requires only local information exchanges between the automata. The learning algorithm has been used for modeling the biological process which recognizes introns from exons in the DNA. It has been applied to medical diagnosis of abdominal pains, and achieves the same performances as a classical expert systems without requiring months for designing the knowledge base. Some variations of the original algorithm are presented. It is shown that the complete behaviour of the network, including the evolution of the weights, can be described using the Lagrangian formalism.
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Configuration of Defective Cellular Array

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Cellular array machines built in the past are SIMD machines with a small number of cells like ILLIAC, or with medium number of simple bit-processors like CLIP, DAP, MPP. With the progress in VLSI technology, MIMD cellular machines with many powerful cells began to appear. When we build a massive cellular array, many cells may be defective. With the adequate computing power in each cell, working cells will be able to configure themselves into desired patterns in spite of defective cells. This paper presents the configuration of cells on the defective cellular array. We proposed an architecture of *massively fault-tolerant cellular array*, and devised self-configuration algorithms. Configuration of cells into linear arrays, binary trees, and two-dimensional arrays are presented. Efficiency of configuration and communication delays have been measured.

Massively Fault-tolerant Cellular Array

The *massively fault-tolerant cellular array* is an array of identical cells which are connected to neighboring cells in various topologies. The cells and the connections with other cells may be defective with high probabilities. Each cell can function as a processing element, as a memory, or as a switching element that connects to other cells. Input and output terminals are connected only at the boundaries of the array.

The three regular interconnection patterns shown in Figure 1 are proposed for the cellular array. The three arrays with the three interconnection patterns of Figure 1 are called *square array, hexagonal array,* and *octal array* respectively. Figure 2 shows a square array with defective cells and defective connections. Note that although the initial array is regular, the ensuing array is not (see Figure 2), as the faults cause breakdown in the regularity of the array.

The computations that the array is intended to perform will determine how the working cells are configured. The logical interconnection of cells for any particular computation can be represented by a graph, called a *computation graph* in this paper. The configuration of cells into a computation graph in the defective array is represented by a graph, called a *connection graph*. For example, the tree in Figure 3(a) is a computation graph; Figure 3(b) shows a configuration of cells into a tree on the defective square array; Figure 3(c) is the connection graph of the configuration. Note that cells represented by dots on the connection graph function as switches.

Configuration of Working Cells

Since our computation model is an array of cells connected only to the immediate neighbors, signals from a cell should be relayed by intervening working cells to a destination cell. For this computation model to be useful, a big cluster of working cells should be formed on the defective array. We used *percolation theory* to study the size and shape of the cluster as the defect density varies. We could guarantee that a big cluster of working cells with adequate number of boundary cells appears on the defective array when the defect density is less than some critical value. The critical value depends on the interconnection pattern of cells.

We devised self-configuration algorithms to configure the working cells into linear arrays, trees, and two-dimensional arrays of cells. We measured the efficiency of the configuration with various defect densities on square, hexagonal, and octal array. The efficiency of the configuration into a graph G, e_G, is defined as

$$e_G = \frac{\text{number of cells used as computation cells}}{\text{number of working cells in the cluster}} \times 100.$$

Figure 4 shows the efficiency of the self-configurations on 120 by 120 array. The self-configuration algorithms, extensive simulation data, and details of the architecture can be found in the reference.

Reference

"Massively Fault-tolerant Cellular Array," Myoung Lee and Gideon Frieder, Proc. 1986 Int'l Conf. on Parallel Processing.

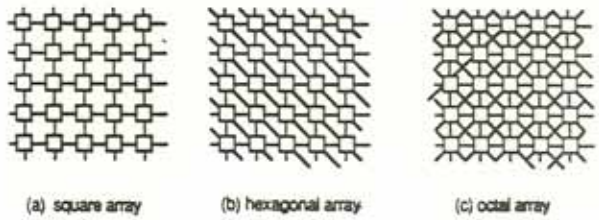

Figure 1. Interconnection patterns of the cellular array

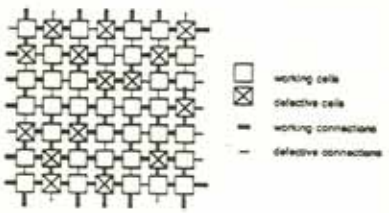

Figure 2. A defective square cellular array

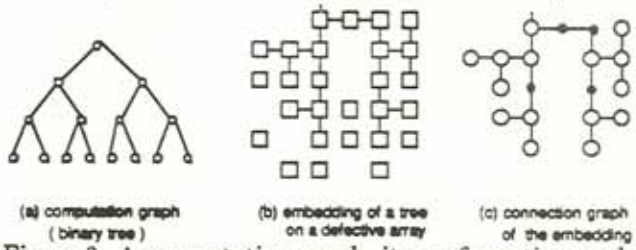

Figure 3. A computation graph, its configuration, and a connection graph

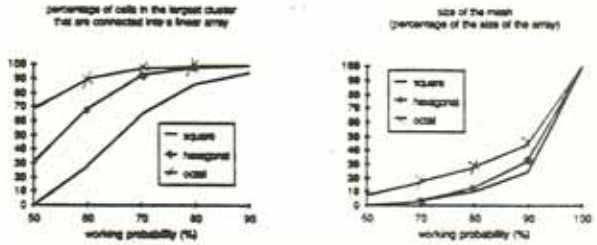

Figure 4. Efficiency of self-configuration

Spectra of Cellular Automata

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and
Stephen Wolfram (*Center for Complex Systems Research, University of Illinois*)

The power spectra of one-dimensional cellular automaton configurations have been investigated. The sequence of site values in a CA configuration is considered as a "signal", whose power spectrum is found by a Fourier transform. The spectrum is related to correlation functions between site values.

Disordered configurations, in which sites take on all possible values with independent, equal, probabilities, give "white", frequency-independent, spectra. Irreversible cellular automaton evolution can lead to the development of structure in the spectra.

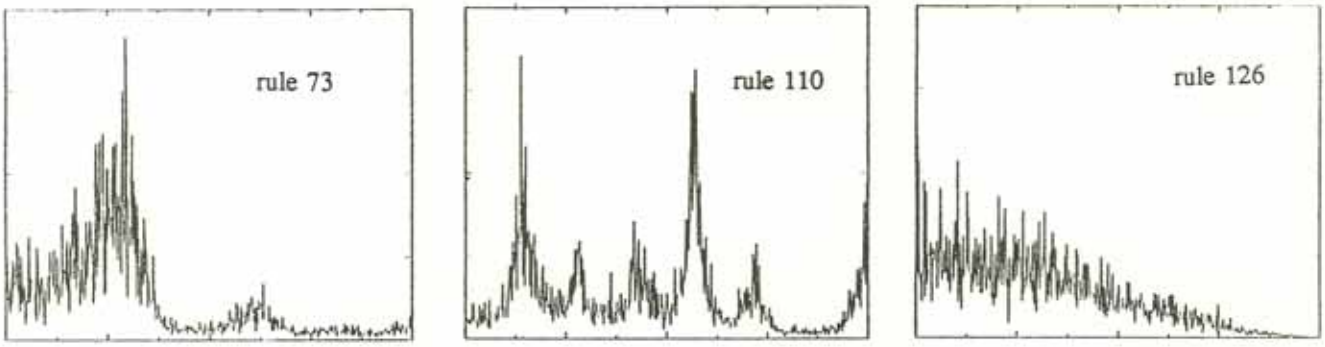
An extreme case is a "perfect crystal", consisting of an infinite periodic repetition of a fixed unit element. Such configurations lead to discrete, delta function, spectra.

Many cellular automata evolve to configurations which can be considered to be made up of a sequence of "domains". Some of these domains may show perfect crystalline order, while others, after suitable blocking transformations, are completely disordered. Many cellular automaton spectra show peaks corresponding to crystalline phases, with widths governed by domain sizes. Usually these peaks are superimposed on a continuous background. In some cases, the background has a roughly Brownian form, with $1/f^2$ frequency dependence. In a few cases, there is some evidence for a $1/f$ component.

Some features of cellular automaton spectra may be obtained by a Markovian approximation. Formal language methods yield minimal finite graphs which represent sets of configurations reached after many time steps. Different phases may be identified as paths through subgraphs. Crystalline phases correspond to purely cyclic subgraphs, whose periods yield the characteristic length scales of the phases. (The set of such periodicities may be related to the zeta function for the system.)

For class 1 and 2 cellular automata, the spectra obtained depend on the statistical properties of the initial configurations chosen. For chaotic cellular automata, it appears that an "equilibrium" is reached whose statistical properties are essentially independent of any details of the initial conditions.

The spectra of two-dimensional cellular automata are expected to show some novel features. In particular, certain two-dimensional cellular automata should yield invariant configurations which show quasi-crystalline order.



Power spectra of some $k=2$, $r=1$ cellular automata (plotted against frequency on a linear scale). The peak in the spectrum for rule 73 occurs at a wavelength of 6 sites; the first peak in the rule 110 spectrum occurs at a wavelength of 14 sites.

Regular language graph for invariant configurations under rule 73. The length 6 cycle in this graph corresponds to the "crystalline phase" responsible for the peak in the power spectrum.

THE FREE ENERGY CONCEPT IN CA MODELS OF PHASE TRANSITIONS

By Daniel G.Maeder, University of Geneva Physics Section, CH-1211 GENEVE4

SUMMARY: The ultimate goal of this work is the simulation of solid-solid phase transitions such as, for example, the thermoelastic martensitic transformation of shape memory alloys. In order to describe their characteristic features of mechanical and thermal hysteresis, a cellular automaton model must take care of external variables, in particular the temperature T and one or several applied stress components XS. Current phenomenological theories of the martensitic transformation are based on the concept of free energy F(e,T) being a function of a continuous variable e called "order parameter", representing the transformation strain. As shown by FALK [1],even a one-dimensional model of this kind explains global behaviour of martensites quite satisfactorily although microscopic aspects (phase boundaries!) are neglected. It is hoped that a CA approach may stimulate understanding the mechanism of discrete domain formation and disappearence. In fact, the nucleation problem is still a matter of discussion among experts [2].

Locally, the strain does not vary continuously but can only switch between a few well-defined values, say ei. This suggests a CA model representing the isotropic (or "austenite") phase by the zero cell state, and the differently strained martensitic variants by cells in nonzero states. Ideally, the CA computer should produce output resembling directly the physical patterns seen in the microscope; the figure to the left displays a hypothetical example showing the different directions of interfaces in all martensite twins (AB,AC,etc.) as well as between the 4 martensite variants and austenite, following ref.[3]. This figure corresponds to an intermediate temperature; the CA must perform the transitions to"all austenite" or"all martensite" states at high and low temperatures, respectively. The particular pattern depends on the presence of lattice defects, so-called "germ" cells, and must be approximately reproduced in successive thermal cycles, with increasing hysteresis at lower temperatures where the patterns should tend to simplify. Furthermore, the CA must respond to applied stresses (XS1,XS2,...) by changing the equilibrium between A,B,C,D cells.

```
CD:::::::::::AA:::::::::::
CCD:::::::::AABD:::::::::::
CCCD::::::::AABBDD::::::::::
:CCCD::::::AABB::DD::::::::
::CCCD::::AABBAC::DD::::::::
:::CCCD::AABBAACC::DD::::::
::::CCCD:ABBAAACCC::DD::::::
:::::CCCDBBAAAACCCC::DD:::::
::::::CCCCACDDDBBBB:::DD::::
:::::::CCC::CDDBBB:::::DD:::
::::::::CCC::CDBB:::::::DD::
:::::::::CCC::CB:::::::::DD:
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So far,suitable transition rules were established for 3-state CA only, modelling either the AB or the AC martensite twins in an austenitic majority environment. The basic considerations were presented- apparently for the first time - at the Lissabon Workshop [4], with emphasis on analogies with LANDAU-type theories. For a cell in state X(=-1,0,1),the free energy analog is F(X,T)=|X|*T+> (Interface energies) + X*(Y1*> X -XS) + |X|*Y2*> |X| where the first sum is over the neighbor cells, the other sums are over the whole lattice, and Y1,Y2 are adjustable positive constants. F is evaluated for X=-1,0,1, and X producing the lowest F is adopted as the new cell state value. For the AB model, reasonable results can be obtained even from a 1-dim CA, in which case the construction of the transition rules involves only 3 independent interface energy parameters (additional terms, not used in ref.[4], are then required to produce the correct hysteresis).

Demonstration programs running on any IBM-compatible PC are available, allowing the following points to be studied:

* the checkerboard problem and various solutions to it
* the nucleation potential of different germ configurations
* rule checking: place test cell at different sites, change its state
* parameter adjusting: watch effect of changes on pattern evolution; etc.

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[2] G.GUENIN and P.F.GOBIN, Metallurgical Trans.Vol.13A,1127-34(1982)
[3] T.SABURI and S.NENNO,"Shape Memory Effect and Related Phenomena":Proc. of an int.conf.on Solid-Solid Phase Transformations,Pittsburgh 1981
[4] D.G.MAEDER,"CA Representation of LANDAU-Type Phase Transform.Theories" Workshop on Cellular Automata,Inst.de Fis.e Mat.,Lisboa,16-18 Dec.1985

PARTITIONING CELLULAR AUTOMATA

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In order to construct a reversible, particle-conserving automaton that simulates Fredkin's billiard-ball model of computation, the idea of a partitioning CA[1] was developed. This idea has proved useful in a number of contexts ranging from hydrodynamics[2] and optics modeling, to rules which support compact digital logic simulations.

CA are usually described by giving the new state of a *center* cell as a function of its nearby neighbors. An alternative approach is to make the new state of a *group* of cells be a function of those cells. We partition the space into small groups of neighboring cells in a uniform manner, and then apply the same rule to all groups. We partition the cells into different groups, and apply another rule everywhere. After going through some cycle of partitions, we start the process over again.

We will describe some interesting CA systems based on a partitioning into 2×2 blocks. Properities such as scale invariance (of a particular type) rotating a rule through a 45 degree angle, reversiblity, conservation laws, sources and sinks, simulating gases and wave phenomena, simulating digital logic, and the development of nonequilibrium structures will be demonstrated.

OTHER TOPICS

The end of the world (collapse of a false vacuum) and other amusements involving second-order reversible CA will be demonstrated. Also a way to rotate a configuration in n-dimensions through a 90 degree angle in log time (in the size of the space) using CA rules on CAM-like machines.

[1] N. Margolus, "Physics-like models of computation," *Physica* 10D (1984), 81–95.

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Direct measure of viscosity in a lattice gas model.

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G. Zanetti

The James Franck Institute, The University of Chicago

We present a method for the direct measure of viscosity in the 2-D F.H.P. lattice gas model.[1]

Our "instrument" is essentially a 2-D Poiseuille viscometer, except that instead of using a pressure gradient to drive the fluid through the channel we use the equivalent, for the c.a. gas, of a uniform body force. This allows periodic boundary conditions along the direction of flow and, together with a way to handle no-slip boundary conditions particularly adapted to this kind of simulation geometry, it permits us to obtain a reasonable measure of viscosity even with systems of very small size (after averaging in space and time to smooth the velocity profile).

The results that can be obtained in this way are good, provided that the Reynolds number and Mach number are kept in reasonable ranges. The velocity profile is parabolic and the density is uniform in the system. In fig. 1 we show the velocity profile obtained from a simulation on a small system. In fig. 2 we demonstrate a comparison between the dependence of the viscosity on the density predicted by the kinetic theory[2] (solid line) and the values directly computed from simulations. One of the interesting possible applications of this method is in the study of the theoretically predicted[3] divergence of viscosity with the size of the system.

This work was supported by ONR and by a donation of computer equipment by IBM. We wish to thank Profs. A. Crewe and L. Kadanoff for helpful advice.

Fig. 1

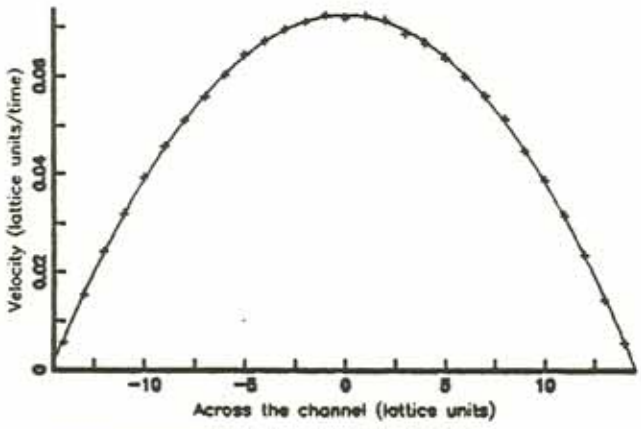
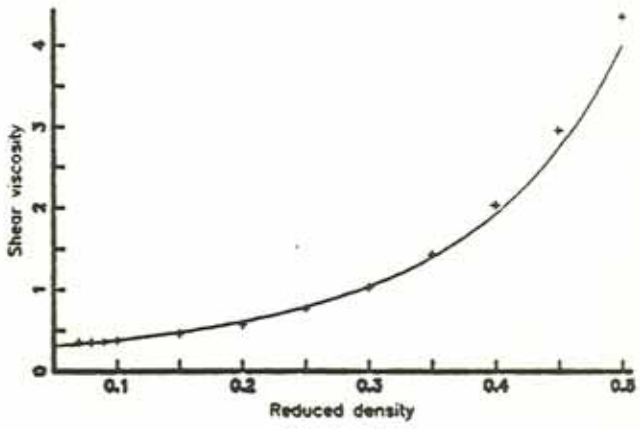


Fig. 2



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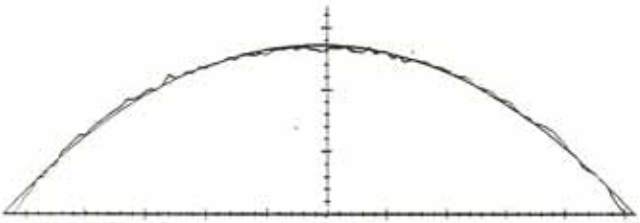
[3] D. Forster, D. Nelson, M. Stephen Phys. Rev. A 16 732 (1977)

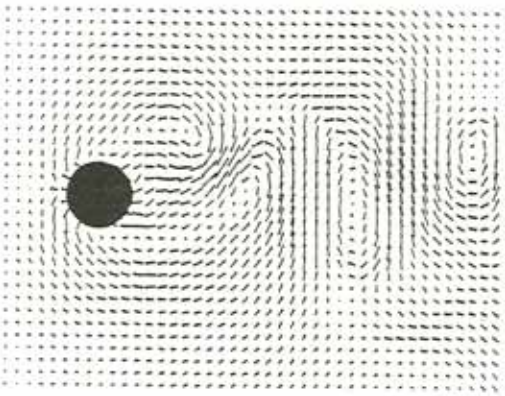
Cellular Automaton Hydrodynamics

Bruce Nemnich (*Thinking Machines Corporation*)
and
Stephen Wolfram (*Center for Complex Systems Research, University of Illinois*)

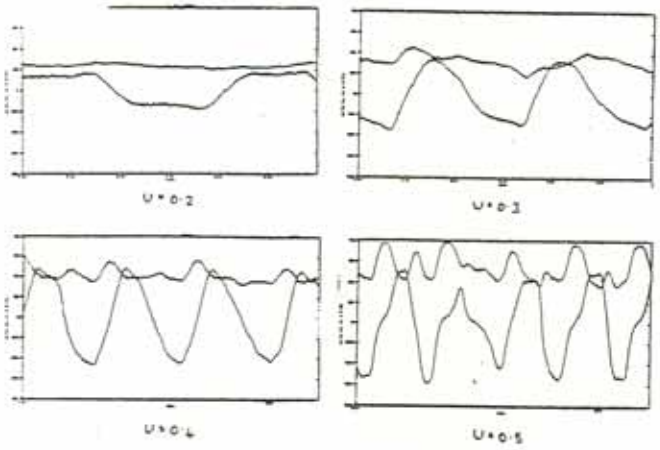
The theory and phenomenology of cellular automaton fluids have been studied. Simulations of two-dimensional cellular automaton fluids have been carried out on a 65536 processor Connection Machine computer. Flows at Reynolds numbers of a few hundred have been obtained. Detailed studies are underway of flows in channels, and flows past simple geometrical objects, such as circular cylinders. Preliminary results indicate good agreement with experiments and existing calculations. At the highest Reynolds numbers investigated, flows past cylinders are observed to make a transition to aperiodicity, corresponding to weak turbulence.



Velocity profile at equilibrium for a cellular automaton fluid in a channel. The smooth curve gives a parabolic fit.



Velocity field around a cylinder moving in a cellular automaton fluid. A 4096×4096 site lattice is used; each velocity vector is obtained by averaging over a 100×100 site region. The overall flow corresponds to the beginning of a vortex street.



Velocity as a function of time for flow behind a cylinder in a cellular automaton fluid. The successive graphs are for increasing Reynolds numbers. The first three cases correspond to regular vortex streets; in the fourth case, aperiodic flow is observed.

New Universality for Stochastic CA-models of Disordered Excitable Media.

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-- Problem background:
Examples of excitable media occur in a variety of natural systems, ranging from heartmuscle or neural membranes to infections or star formation. One important question is when and how excitation propagates through the medium. With local interactions and an effectively stochastic dynamics, one finds that the propagation under marginal conditions shows non-trivial collective fluctuations, just like in critical behavior of statistical physics systems.

-- SCA models:
The analogy can be made explicit by writing down a family of Stochastic Cellular Automata models with 2 local states $s_i \in \{0,1\}$, z short-range couplings $c_{ij}>0$ and time-invariant, local state transition probabilities $P(s_i(t+1) = F_i(\sum_j c_{ij} \cdot s_j(t))$, with $F_i(x \leq 0) = 0$ and $0 < F_i(x>0) < 1$.
The restrictions on F(x) represent the single deterministic rule causing the "vacuum" ($s_i=0 \; \forall i$) to be the unique absorbing state of the process.

-- Special cases reduce to known models:
Imposing i-independence and i-j symmetry yields models that, when specified in D dimensions, generate clusters of 1's in (D+1)-spacetime that are exactly equivalent to those of Directed Percolation on (D+1)-lattices. Thus, known universal exponents describe the critical behavior in the restricted models.

-- New universality, multicriticality and slow phases:
In contrast, the general models have quenched spatial disorder specified by $F_i(x)$ and c_{ij} depending randomly on i and j, modelling inhomogeneous media. A Harris-type argument shows that the disorder is incompatible with DP-class exponents. In fact, Monte Carlo-simulations in D=1 and 2 of many SCA models in this family show the existence of a new universality class, characterized by exponents that are quite different from the DP-values. In addition, with "dilution"-type disorder, the phase diagram contains a multicritical point and the SCA-analog of a Griffiths-phase having non-exponential relaxation.

2-d LATTICE GAS CELLULAR AUTOMATA COMPRESSIBLE FLOWS

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We have used the Frisch-Hasslacher-Pomeau (FHP) cellular automaton to compute macroscopic hydrodynamic flows for typical geometric configurations. In the FHP model, "particles" move with unit velocity in any of the six directions of a triangular lattice and collide elastically when they meet at a lattice node. The collision rules guarantee particle number conservation and total momentum conservation and obey the exclusion principle that two particles with the same velocity cannot occupy the same node. With these simplifications, the state of the lattice can be represented by an array of Boolean variables identifying the presence of particles on the links between nodes, and all flow calculations are reduced to unit displacements in memory and logical operations at the lattice nodes. Hydrodynamic quantities are obtained by averaging particle number and velocities over a few tens adjacent nodes ; they are found to obey Navier-Stokes equations for low speeds.

Linear hydrodynamic properties of the model system have been studied by measuring the response of the system to a periodic perturbation to determine the sound velocity and the fluid viscosities. Flows around obstacles have been studied for various obstacle shapes. Obstacles are represented by decomposing their boundary into lattice links and imposing special collision rules on the corresponding nodes so that the component of the velocity normal to the boundary be zero. We have computed flows behind plates, into air intakes, around cylinders and wings and obtained boundary layer profiles. Flows in a pipe with a sudden expansion have also been measured. A constant flow is obtained by imposing a non-zero mean flux of particles at the input and the output of the pipe while ensuring total flux conservation. The Reynolds number of the problem is fixed by the step height and the flow velocity. At moderate Reynolds numbers, a stationary vortex forms behind the expansion wall. We measured the flow reattachment point on the boundary (defined by zero mean longitudinal speed) for various geometries and several Reynolds numbers. At the initial time, the flow has a Poiseuille velocity profile and the fluid behind the wall is at rest (fig.1). The system then evolves according to the cellular automaton rules. When stationarity has established, longitudinal and transverse velocity profiles are measured by averaging over layers a few nodes wide. Pressure maps demonstrate the compressible nature of the flow. The results are compared with experimental and other numerical data and will be presented in graphical form. A typical example is shown in Fig.2.

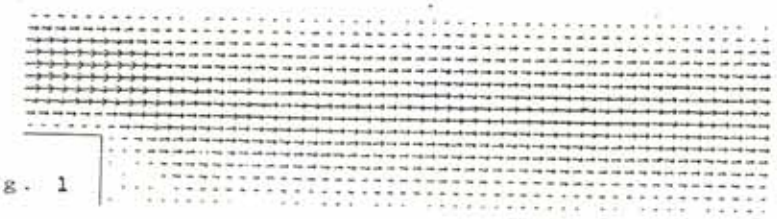

Fig. 1

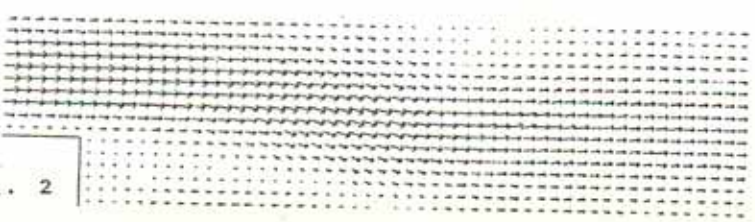

Fig. 2

Deterministic Lattice Models for Interface Dynamics

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We present a new model for the temporal evolution of interfaces in two dimensions. The model interface may represent many different physical phenomena, including fluid interface dynamics (in the Hele-Shaw configuration), solidification, and diffusion limited aggregation. The model is a hybrid that uses both discrete and continuous elements. The state space of the model is the space of configurations of two variables over a two dimensional lattice: a continuous variable that evolves in time according to a numerically stable approximation to the continuum diffusion equation, and a discrete variable that evolves according to a simple rule that couples nearby site values and the contiuum field.

Illustrated is a sequence of macroscopic forms for an interface, obtained by applying the dynamical rule to a small random initial seed. The diffusion length, a parameter in the model, is varied to obtain the sequence. (a) Large diffusion length: a form that shows no anisotropy, and that has a fractal dimension between 1.6-1.8. (b) Intermediate diffusion length: a form that shows weak anistropic effects, and that has a dimension of 2.0. (c) Short diffusion length: a form that shows strong anistropic effects, with a stable parabolic tip growing at constant velocity and shedding sidebranches periodically.

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GLOBAL PROPERTIES OF FINITE CELLULAR AUTOMATA

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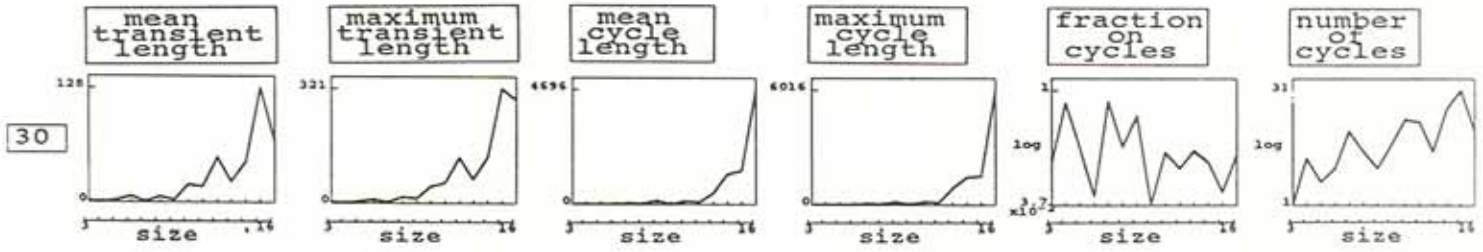
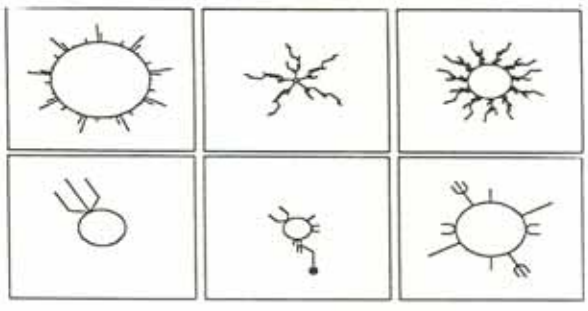
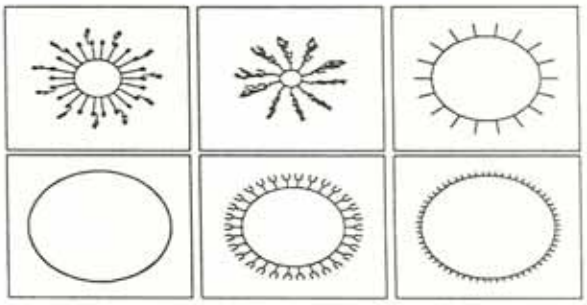
How does the behavior of a binary, one-dimensional, nearest-neighbor cellular automaton change with as register size changes? Does behavior approach an asymptote as size becomes large?

A computer program was used to investigate the global behavior of all 256 possible simple CA rules within a regester of finite width N, for N ranging between 3 and 16. The cycle and transient structure was calculated for each rule, and some fundamental properties plotted as a function of N.



The cycle structure is plotted as a state transition diagram, in which a cycle is plotted as a circle, and a transient leading to that cycle as a straight branch leading away from the cycle.





SIMPLIFIED CELLULAR-AUTOMATON SOFTWARE FOR IBM PERSONAL COMPUTERS

By Charles Platt
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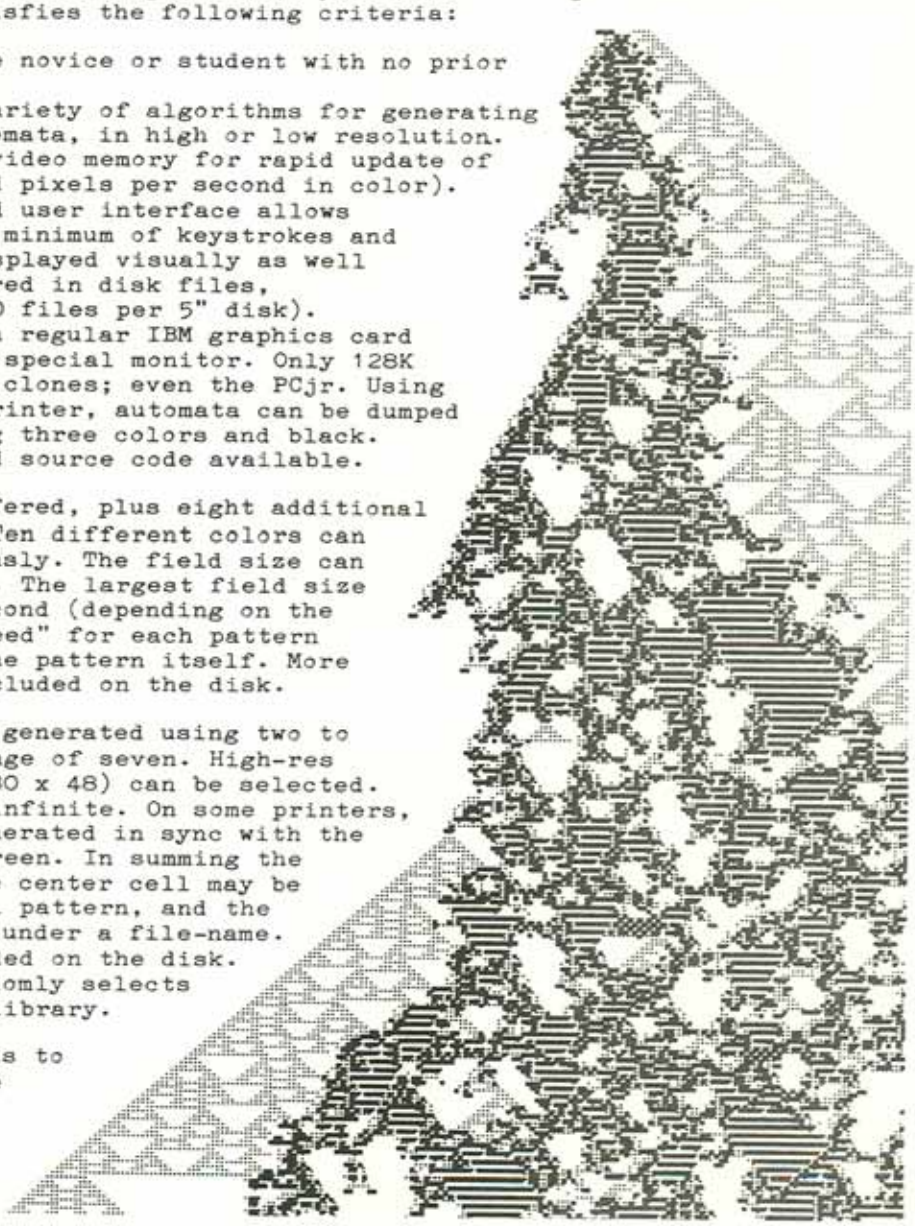
Most software for generating cellular automata is not "user-friendly" and may require special equipment or large amounts of computer memory. I have developed new software for educational purposes, which satisfies the following criteria:

1. Simple. Can be used by the novice or student with no prior knowledge of cellular automata.
2. Flexible. Offers a wide variety of algorithms for generating linear (and other) cellular automata, in high or low resolution.
3. Fast. Directly addresses video memory for rapid update of patterns (up to 13,000 displayed pixels per second in color).
4. Convenient. Highly refined user interface allows modification of patterns with a minimum of keystrokes and figuring. All parameters are displayed visually as well as numerically. They can be stored in disk files, retrieved, and edited (about 100 files per 5" disk).
5. Standard equipment. Only a regular IBM graphics card (or equivalent) is required. No special monitor. Only 128K memory needed. Works on all IBM clones; even the PCjr. Using an Epson or IBM or compatible printer, automata can be dumped directly from screen, simulating three colors and black.
6. Easily modified. Annotated source code available.

The Conway "Game of Life" is offered, plus eight additional variants of the usual formula. Ten different colors can appear on the screen simultaneously. The field size can be varied from 9 x 9 to 39 x 39. The largest field size is updated several times per second (depending on the computer's clock speed). The "seed" for each pattern can be stored separately from the pattern itself. More than 100 sample patterns are included on the disk.

Linear Cellular Automata can be generated using two to four colors, selected from a range of seven. High-res (320 x 192 pixels) or low-res (80 x 48) can be selected. Growth can be finite or pseudo-infinite. On some printers, an "endless" printout can be generated in sync with the pattern as it scrolls up the screen. In summing the states of surrounding cells, the center cell may be included or omitted. The initial pattern, and the resulting growth, can be stored under a file-name. Sixty sample patterns are included on the disk. In "auto" mode, the program randomly selects and displays automata from its library.

The intention of this software is to make cellular automata available and comprehensible to the widest possible audience.



Bus Automata: Cellular Automata with Global Dynamic Variability in Choice of Effective Neighbor Sets for Each Cell

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Bus automata (BA's) generalize cellular automata (CA's) by providing a modular global interconnection network, each module controlled by its local automaton, in place of the fixed patterns of nearest neighbor communication characteristic of conventional CA's. Each cell receives or sends information over links with its nearest neighbors and has the option of connecting any of its input links to any of its output links through an internal channel. Any pattern of connected links and channels is a bus; BA algorithms are implemented via dynamic BA architecture changes (successive bus configurations). We have used BA's to explore some ultimate limits of parallel/distributed processing, achieving "immediate" recognition of many complex languages and patterns, and to model visual/nervous systems. Recent work has led to near obliteration of the distinction between BA hardware description languages and the corresponding distributed programming languages, as well as to development of generative grammars for patterns in the plane. Recognition is immediate for many patterns produced thereby (including fractals), and the general case (non-immediate) goes over to full Turing machine generality with parallel speed-up.

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Cellular Automata with the Connection Machine
The Cellular Automata Toolkit

Jim Salem

Thinking Machines Corporation

The Connection Machine (CM) has proven to be a powerful, general purpose cellular automata machine. The CM has been successfully used to simulate many different types of cellular automata of various dimensions. This poster describes work on the CM in three areas : fluid flow simulations, simple one dimensional CA, and CA with dimension > 2. More generally, it describes the Cellular Automata Toolkit developed by the author at Thinking Machines.

The Cellular Automata Toolkit is a CA researcher's toolbox. It contains utilities for creating CA of arbitrary size and topologies and moving them into and out of the CM. CA rules may described using a special user-interface window or may be expressed in SIMPL, a variant of the LISP language developed for the Connection Machine. The CA programmer has available many data collection, analysis and display utilities.

The Toolkit is based on the CONFIGS virtual processing system. This system allows even small Connection Machine Systems to simulate millions of cellular automaton cells. When running the CONFIGS system, a CM can store many different CA configurations at once.

CA with arbitrary interconnections are easily simulated using the CM's high-speed parallel routing network. Topologies simulated on the CM include: grids of any number of dimensions and hexagonal lattices. The CM's boolean processors fit the CA cell size exactly. Thus no computation is wasted on leftover bits as when running CA on computers with long word lengths.

In addition to simulating cellular automata, the CM is also powerful in data collection, display and analysis of CA results. Its parallel processing has been used to gather statistics, area average results, time average results, render 3D CA in meaningful ways, and image enhance results.

The CM has performed well in simulating fluid flow using CA techniques. The CM attains speeds of about 1 billion site updates per second on a 4000 x 4000 hexagonal lattice. We have observed vortex streets and aperiodic flow around arbitrarily shaped objects.

Information Density Near a Phase Transition

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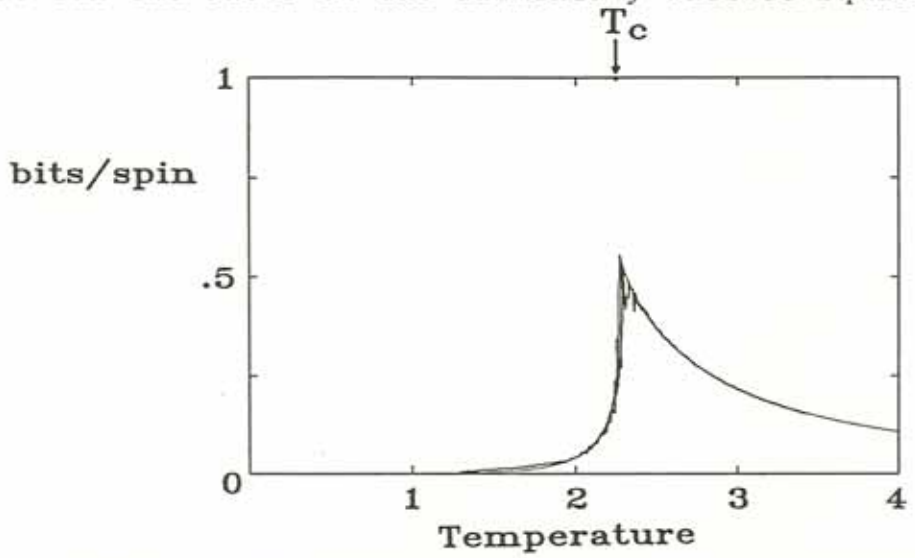
Spatial mutual information measures are used to characterize the degree of structure in the spin patterns of the two-dimensional Ising model near its critical point. The "information density" is seen to be equivalent to a thermodynamic quantity, the difference between the single-spin entropy and the full thermodynamic entropy per spin, $I = H(1) - S$. Any correlations in the spin pattern reduce the thermodynamic entropy from the maximum it could have with no nearest neighbor interactions. This measure displays a sharp but finite maximum at the phase transition. Shown superposed on the figure below are the exact thermodynamic curve, obtainable from the analytic solutions of Onsager and Yang, and the spatial mutual information values, computed from average local conditional probability distributions. The average information density in a two-dimensional geometry is given by $I(AB|CD)$, the mutual information between two sides of the elementary lattice square ABCD.



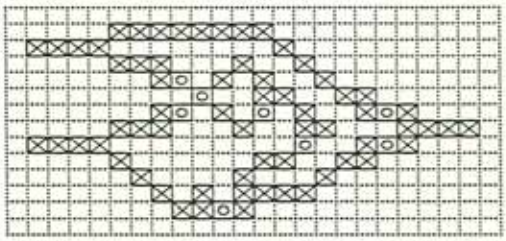
Lattice dynamics were provided by a deterministic cellular automaton rule rather than monte-carlo updates. As shown by Creutz, a suitable deterministic rule with hidden momentum variables can give accurate thermodynamic spin statistics without the necessity of generating random numbers.

The information density is a "stored information" type of statistic, providing an upper bound to the amount of information that can reliably be transmitted into the future using the system as a medium. It is hoped that this statistic will be useful in characterizing the degree of order in spatially extended systems where there is no conserved energy, such as reaction-diffusion equations. Also, this result in the thermodynamic context gives perhaps some credence to the speculation that systems in general have greater information storage capacity near phase transitions.

Cellular Automata Circuits

Brian Silverman
Logo Computer Systems Inc.

It is well known that some celluar automata are computationally universal. This has been proven with several different rules. However, the rules used in these proofs lead to "computers" that are quite large and counterintuitive. There are other rules where logic gates can be built in a rather straightforward way.



The picture above, for example is an binary serial adder in a 7 state per site automata. Two of the states are used to define the signal paths. Four more are compontents of trivial gliders that move along these paths. The seventh is the background.
There are a lot of other ways to build logic circuits using cellular automata. A rule that supports gliders can often use them to transmit information. Stable structures can be used to construct gates. Several examples will be given inlcuding Conway's Life Game and others that allow for much higher logic densities.

Demonstration of a Pipelined VLSI Based Cellular Automaton Processor

Kim Strohbehn et al. (Applied Physics Lab., Johns Hopkins)

A simple and cheap Cellular Automaton Processor has been developed as a coprocessor board for the IBM P.C.. The processor handles cellular logic transformations on a 9-bit neighborhood of a single bit, 256x256 array at about 240 array transformations per second in a pipelined mode. Twelve identical custom VLSI chips, each performing a sequential transformation, form the pipeline stages.

We propose a demonstration of cellular automaton processing using this coprocessor. We will demonstrate Conway's Game of Life, time-reversible operators, the Abingdon cross image processing benchmark, and simulated percolation processes (See Figure 1). Interested observers will have the opportunity to implement algorithms of particular interest to them on the APL coprocessor.

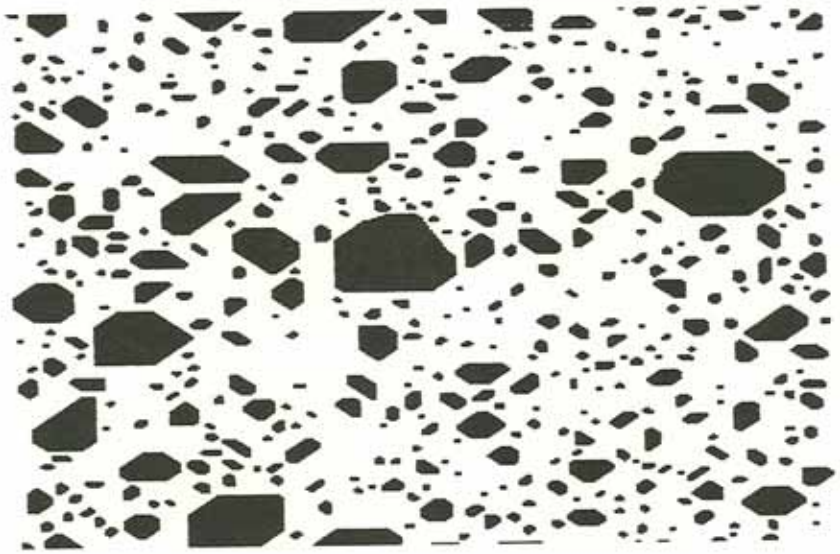


Figure 1. Intermediate image in a percolation simulation.

TIME VARYING CELLULAR AUTOMATA

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The motivation for our work is the modeling of systems which have hierarchical control structures. An example of such a system is the metabolic control in cells.

We define a new class of cellular automata called *time varying cellular automata (TVCA)*. *TVCA* contain the class of elementary cellular automata (*ECA*) investigated by Wolfram in 1983. The basic idea of *TVCA* is to let the *ECA* rules vary with each clock step, under some form of control rule. *TVCA* exhibit a richer class of behaviors than *ECA* while exhibiting a number of important decidability characteristics. Among the *TVCA* is an important subclass called *periodic time varying cellular automata (PTVCA)*. In this subclass, the set of control rules varies periodically with time.

Our poster demonstration will provide examples of *TVCA* and *PTVCA* and will exhibit the following results:

1. Equivalence classes of *TVCA* with *ECA*. This demonstrates the fact that many different control strategies give rise to identical behavior. An example of this is *ECA(90)*, the elementary cellular automata under the Rule of 90, which is found to be equivalent to *PTVCA(2;x,y)*, the periodic time varying cellular automata of period 2 with x being a member of the set of *ECA* rules (18, 50, 90, 122, 146, 178, 218, 250), and y being a member of the set of *ECA* rules (18, 90, 146, 218).
2. Definition of *PTVCA* in terms of formal rewriting systems. Matrix grammar and control grammar rewriting schemes establish a well defined procedure by which larger classes of legal configurations may be described. The relationship between linear bounded cellular automata, normal forms of productions in grammars and hierarchical control structures is explored.
3. Algebraic properties of *PTVCA* relating to category theory.

Evidence that quantum fields are no cellular automata – summary

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1. From a topological point of view, cellular automata are equivalent to their associated *dual lattices*. The dual lattice is gained by shrinking a cell to point size, while keeping its connections with surrounding cells. In that way, results from lattice theory can be applied to cellular automata theory and *vice versa*.

2. A no–go theorem[1] states, that for quite general classes of local lattice models with [locally defined] point charges, there can be no unitary, invariant theory on the lattice without equal numbers of left– and righthanded fermions for each combination of quantum numbers. This is *not* the phenomenology seen; for instance in the weak interactions, there is no right– and lefthanded symmetry associated with the hypercharge states $Y(e_R) = 1$ and $Y(e_L) = \frac{1}{2}$.

3. There may be ways to circumvent the dilemma stated by the no–go theorem; three of those are listed below:

(i) to give up locality of the transition law[2], such that non-neighboring cells contribute. This can be discussed in the framework of the theory of finite elements;

(ii) to give up the local definition of quantum numbers, which then appear as "smeared out" charge sources etc.;[3]

(iii) "dimensional shadowing", a technique for projecting higherdimensional manifolds onto lowerdimensional ones.[4]

4. The topological equivalence between cellular automata on the one hand, and their dual lattices on the other hand, allows application of theorems established for either one of those to the other one. For instance, it can be inferred that a wide class of local field theories on the lattice [those with more than one states in two space dimensions] are universal computers.

The author acknowledges discussions with Anton Zeilinger. This work was supported by BMWF, project number 19.153/3–26/85.

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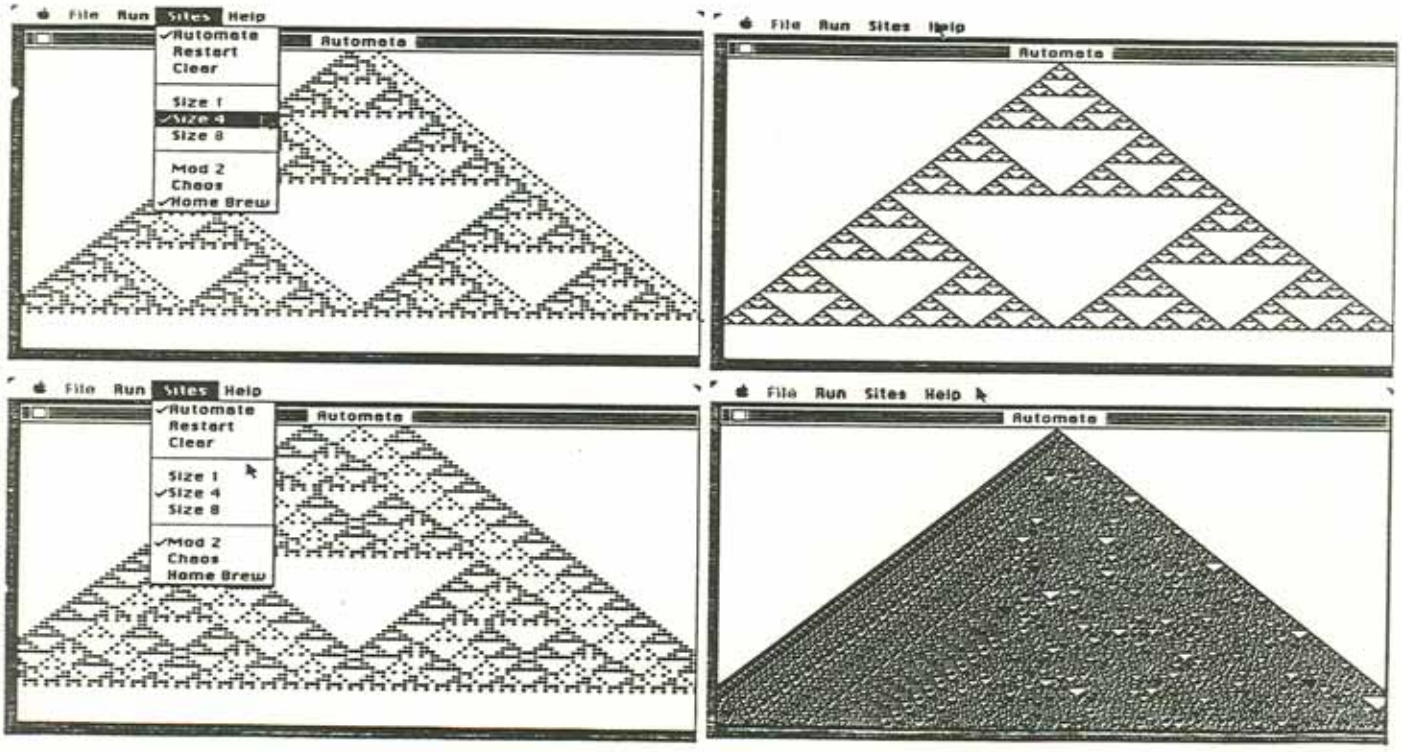
Cellular Automata as Pop Art

N. Tufillaro, J. Reilly. Physics Department, Bryn Mawr College, Bryn Mawr, PA. 19010, and R. Crandall. Reed College, Portland, Oregon.

Wolfram recently suggested that CA is a fertile medium for pop art. CA's most striking feature is the ease with which the user can create a myriad of unique, intriquing and captivating patterns. We developed a popular CA program -- AUTOMATA -- that features a simple user interface and allows the artist to explore rules and initial conditions by pointing and clicking. The design and user-interface of Automata will be described with an eye towards defining standards for the user interface and core function calls of future CA systems designed for the general public.



BOOLEAN CALCULUS ON CELLULAR AUTOMATA

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This poster presents applications of the Boolean derivative[1] to cellular automata.

A Boolean cellular automaton with N cells is a fully discrete dynamical system whose evolution is given by the iterations of a **global mapping** F

$$F : \{0,1\}^N \mapsto \{0,1\}^N.$$

The homogeneity and the locality of cellular automata permit a compact definition of F in terms of a **local transition rule** f

$$f : \{0,1\}^n \mapsto \{0,1\}$$

that maps the occupancies of the **neighborhood** (of size n, with $n \ll N$) to the next value of the center cell. Since the rule f is fully described by a table look-up of length n, all its properties can be readily obtained by simple inspection of this table.

Deriving properties of F from those of f is in general a difficult problem. The Boolean derivative f' of f is a nontrival property that can be used for characterizing F. Indeed, the Boolean derivative F' of F can be defined as the $N \times N$ matrix

$$F' = \begin{pmatrix} (\ f'\ ) & & 0 \\ & (\ f'\ ) & \\ & & \ddots \\ 0 & & (\ f'\ ) \end{pmatrix}$$

where f' is the $1 \times n$ row matrix $f' = (\frac{\partial f}{\partial x_1}, \ldots, \frac{\partial f}{\partial x_n})$, defined[1] with

$$\frac{\partial f}{\partial x_i} = f(x_1, \ldots, x_i \bigoplus 1, \ldots x_n) \bigoplus f(x_1, \ldots, x_i, \ldots x_n)$$

where the x's are the cell values (0 or 1).

A matrix element F'_{ij} is one if varying site j at time t affects site i at time $t+1$, it is zero otherwise (cf. the 0's away from the diagonal). The matrix F' has the following "good" properties that one can expect from a derivative:

- F' vanishes if the rule f is constant;
- $F' = F$ if F is linear (i.e., f is the Exclusive OR of some neighbors). Hence F' extracts the linear part of a general nonlinear global mapping F;
- The Jacobian $\det|F'|$ does not vanish if the rule is invertible (inverse function theorem on cellular automata).

The notion of Boolean derivative thus permits the construction of an elementary Boolean calculus on cellular automata; it should lead to fruitful connections between cellular automata and continuous systems.

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[1] See e.g., the books by A. Thayse (Springer, 1981) and by F. Robert (Springer, 1986).

LEARNING PHYSICS WITH CELLULAR AUTOMATA

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Fully discrete models lend themselves to exact simulation by digital computers. By avoiding the usual detour through calculus and numerical analysis, these models can focus on deep principles and subtle effects with very economical conceptual means.

Cellular automata are a type of fully discrete dynamical systems with physics-like behaviors. They provide an interactive environment by which one can experiment with notions such as:

- Micoscopic irreversibility and self-organization (prebiotic evolution), where entropy *decreases*, in contrast with

- The second law of thermodynamics, where microscopic reversibilty entails macroscopic irreversibilty (the arrow of time). There, the detailed entropy is *constant*, but the measurable coarse-grained entropy *increases*. Cellular automata dynamics offers easy and spectacular implementations of the paradoxes of Loschmidt (reverse all velocities) and of Zermelo (make a smaller synthetic universe, with a shorter Poincaré recurrence time).

- "Advanced" concepts of ergodic theory, e.g., ergodic surface, metric transitivity, microcanonical ensemble, order parameter. Cellular automata can also show how some physical systems (e.g., amorphous hard alloys, glasses, spin-glasses) are *not* ergodic.

- Long survival of metastable phases (supercooled vapor, superheated liquid), a phenomenon relevant to phase-transitions via nucleation, Wilson and bubble chambers, fog, and how a tea-kettle works.

- Light-cone and causality principle in relativity (these occur in physics and in cellular automata models for the same reason: information propagates at finite speed). Supralumimal propagation of noncausal waves (phase-velocity vs. group-velocity).

Continuous time cellular automaton simulator

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I plan to demonstrate the operation of a 6X9 continuous time array and present a poster review of the same. The array is programmed via a suitably interfaced IBM-PC. Neighborhoods are octal, and all mappings take this eight element neighborhood vector to a scalar at the central site. The average rate of bit transition over the array is 250 KHz.

Rules which yield isotropic time average behavior will be demonstrated. Such rules yield periodic, quasi-periodic, and chaotic time domain activity. This activity is analyzed with a Nicolet Instruments 50 nanosecond digital analyzer available through Brown University. With it, exact time domain display of chaotic behavior is facilitated. In addition, the Nicolet is capable of computing and displaying power spectra.

Average values on the lattice when boundary conditions are actively impressed are displayed in real-time on an oscilloscope. The impressed boundary condition is variable in both frequency and average value, and the effect of changing these parameters is easily seen. Since spatial gradients in this value fit exponential functions nicely, this demonstration should be of particular interest to the physical scientists.

The poster will discuss the order and chaos seen on the array and the prospects for the dynamic modeling of analytic functions. It will also extend formalism to the demonstration. Of course, it will push the merit of continuous time processes.

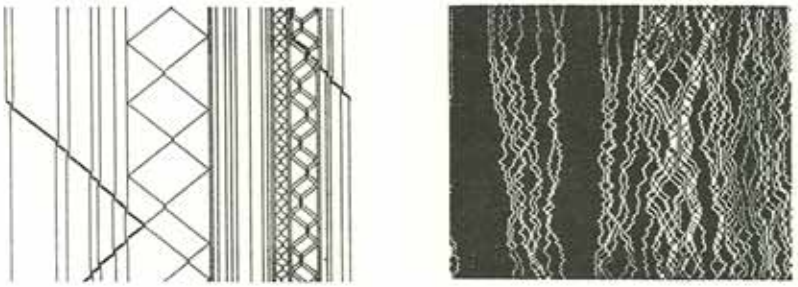
Minimal Cellular Automaton Approximations to Continuum Systems

Stephen Wolfram (*Thinking Machines Corporation and Center for Complex Systems Research, University of Illinois*)

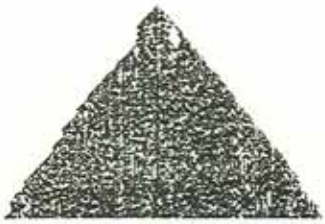
Cellular automata can show continuum behaviour for much the same reasons as physical systems consisting of discrete elements such as particles do. It is necessary that additive conservation laws exist, so that macroscopic quantities such as density can be defined. In addition, it is necessary that the microscopic details of configurations become random on a short time scale, so that statistical averages are accurate. This second condition corresponds to the validity of the Second Law of thermodynamics.

For both theoretical and practical purposes, it is worthwhile to seek the simplest cellular automaton rule whose large scale behaviour has a particular continuum form. It appears that such a rule is best found not by construction, but by explicit searching or by an iterative or adaptive procedure.

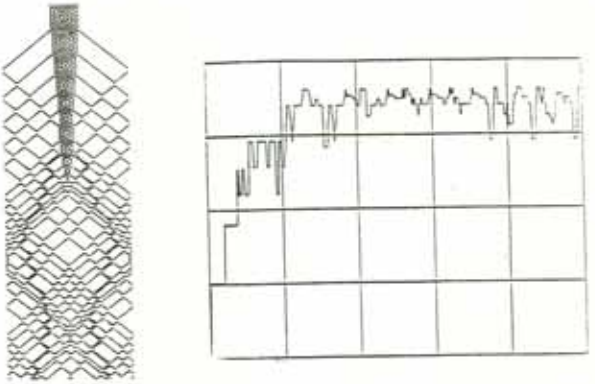
I have searched for the simplest cellular automaton which reproduces the diffusion equation in one dimension. The result is a $k=3$ rule, in which the mapping at each time step depends only on pairs of adjacent sites. The rule is invertible, and conserves the number of binary bits in each configuration. Although the rule is entirely deterministic, it appears to act as a good pseudorandom sequence generator, and to yield "particle trajectories" which are effectively random walks.



Microscopic diffusion at two densities in the minimal cellular automaton approximation to the one-dimensional diffusion equation.



Pattern of differences produced by a single site initial change. The microscopic configuration of the system is unstable to small perturbations.



Randomization of the microscopic configuration, starting from a simple initial condition, together with the corresponding coarse-grained entropy increase.

SIMULATION OF BIOLOGICAL AND SOCIAL SYSTEMS WITH CELLULAR AUTOMATA

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Lawrence Livermore National Laboratory
Livermore, California 94550

1. Simulating the Color Patterns on Vertebrate Skin

   a) Diffusion processes are believed responsible for biological patterns.
   b) Complex patterns can be generated by giving cells both activation and inhibition properties.
   c) The diffusion equations can be simplified to a non-nearest neighbor cellular automaton.
   d) Variation of the ratio of inhibition to activation strengths yields a transition from spot to stripe patterns.

   e) Directional striping can be obtained from non-circular activation and inhibition zones.
   f) CA models thus mimic closely real biological systems and provide an impetus for new experimental and theoretical research.

2. Simulating the Growth of Cities

   a) Cities grow by an accretionary process at their boundaries, as well as a densifying process at their centers.
   b) Cities also develop different kinds of structures, for example residential, industrial, government, etc.
   c) CA models allow one to test simple algorithms for the growth process and to compare the results of simulations with actual city plans.
   d) Nearest-neighbor CA models in fact show many of the features of city structure.
   e) These models may be used to suggest useful ideas about the origins of the earliest cities, and to generate hypotheses about collective human behavior.

Cellular Automata '86: Provisional Program

Posters, computer demonstrations and other presentations will be set up throughout the conference. Each poster will carry two cards which give times when the poster authors should be on hand to discuss their work.

Various tutorials and discussions have been arranged. It is expected that each one will be of interest only to a fraction of conference participants. Additional discussions, talks, etc. should be arranged by conference participants. There is a board near the entrance to the main conference area on which announcements of such events should be posted. See the "Notes on Discussions".

Most of the tutorials and discussions scheduled so far will be held in the main conference lecture room (and can be watched on closed-circuit TV in the main conference area). Additional meeting rooms for smaller groups are available upstairs.

Sunday, June 15

6:30 pm	Meeting area opens for preparation of presentations

Monday, June 16

8:30 am	Registration begins
9:00	Coffee
10:00	Tutorial: History and background of CA (Tom Toffoli)
11:15	Tutorial: Basic CA science (Stephen Wolfram)
12:30	CA '86 group photograph
1:00 pm	Lunch
2:30	Discussion: CA models for pattern formation
3:00	Tea
3:30	Discussion: CA models for fluids
4:30	Discussion: CA simulation software
6:00-8:00	Barbeque at Thinking Machines Corporation (245 First St., Cambridge) (See map and other information on separate page.) Including Connection Machine demonstrations.

Tuesday, June 17

8:30 am	Coffee
9:00	Tutorial: CA as dynamical systems (Doug Lind)
10:00	Discussion: CA mathematics
11:00	Discussion: CA models of the brain
12:00	[To be announced]
1:00 pm	Lunch
2:15	Tutorial: Statistical physics and CA (Charles Bennett)
3:00	Tutorial: Computation in probabilistic CA (Peter Gacs)
3:30	Tea
3:45	Discussion: Probabilistic CA
4:45	Discussion: CA for art and design
5:45	Discussion: CA in education
7:00	Banquet in McDermott Court (see map).

Program

Wednesday, June 18

8:30 am	Coffee
9:00	Tutorial: Parallel computation (Danny Hillis)
9:45	Discussion: CA computer architectures
10:45	Discussion: Pattern recognition with CA
11:45	[To be announced]
1:00 pm	Lunch
2:30	Discussion: Future directions and plans for CA '87
4:00	Conference closes (material must be removed by 7 pm)