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AN EXPERIMENTAL ANALYSIS OF PROGRAM REFERENCE PATTERNS
IN THE MULTICS VIRTUAL MEMORY

by

Bernard Stewart Greenberg

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ABSTRACT

This thesis reports the design, conducting, and results of an experi-
ment intended to measure the paging rate of a virtual memory computer
system as a function of paging memory size. This experiment, conducted on
the Multics computer system at M.I.T., a large interactive computer utility
serving an academic community, sought to predict paging rates for paging
memory sizes larger than the existent memory at the time. A trace of all
secondary memory references for two days was accumulated, and simulation
techniques applicable to "stack" type paging algorithms (of which the
least-recently-used discipline used by Multics is one) were applied to it.

A technique for interfacing such an experiment to an operative com-
puter utility in such a way that adequate data can be gathered reliably
and without degrading system performance is described. Issues of dynamic
page deletion and creation are dealt with, apparently for the first re-
ported time. The successful performance of this experiment asserts the
viability of performing this type of measurement on this type of system.
The results of the experiment are given, which suggest models of demand
paging behavior.

THESIS SUPERVISOR: Jerome Howard Saltzer
TITLE: Associate Professor of Electrical Engineering
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# Table of Contents

<table>
<thead>
<tr>
<th>SECTION</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>2</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>3</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td>4</td>
</tr>
<tr>
<td>TABLE OF CONTENTS OF APPENDIX A</td>
<td>6</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>7</td>
</tr>
</tbody>
</table>

## INTRODUCTION
1. Brief Statement of the Problem                   | 8    |
2. Summary of Result                                | 8    |
3. Summary of the Work of This Thesis               | 9    |
4. Structure of This Thesis                         | 11   |

## CHAPTER I Virtual Memory Performance           | 12   |
1.1 Memory Performance Prediction as a Goal        | 12   |
1.2 Program Reference Patterns and Models          | 17   |
1.3 The Experimental Determination of Predicted Headways | 19    |
1.4 Previous Work in this Area                     | 22   |
1.5 Novelty of the Work in This Thesis             | 24   |

## CHAPTER II The Design of the Experiment          | 26   |
2.1 Stack Algorithms and the Extension Problem     | 26   |
2.2 The Extension Problem and Multics              | 31   |
2.3 Performing an Experiment on Multics             | 40   |

## CHAPTER III The Results of the Experiment        | 46   |
3.1 The Conducting of the Experiment               | 46   |
3.2 The Results of the Experiment                  | 47   |
3.3 Reference Probability Models Suggested by these Results | 55    |
3.4 Accuracy of the Reported Results                | 58   |
3.4.1 The Effect of Lost Data
   3.4.1.1 Lost Counter Accuracy 59
   3.4.1.2 Stack Shifting Inaccuracies 60
3.4.2 Global Transparent Paging Device Inaccuracies 61
3.4.3 Inaccuracies Resulting from List Deletions 64
3.4.4 Other Inaccuracies 67
3.5 Our Result and the Linear Model Measurements 72

CHAPTER IV Conclusions and Suggestions for Future Research 74
   4.1 Conclusion 74
   4.2 The Paging Model Suggested 77
   4.3 Unanswered Questions and Future Directions 78

APPENDIX A A Structured Program Description of Multics 79
   Page Control 83

APPENDIX B Implementation of the Hardcore Meters 83
   Interface Details 127

APPENDIX C System Performance Graphs During Experiments 131

BIBLIOGRAPHY 139
# Table of Contents of Appendix A

1. A Brief Overview of Page Control .......................... 85
2. An Explanation of the Language Used to Express This Description .......................... 88
3. A Top-Level Programmatic View of Page Control .......................... 94
4. A Top-Level View of the Objects Used by Page Control .......................... 96
5. A Description of the Object Types Used in Page Control .......................... 98
6. The Global Variables Used by Page Control .......................... 101
7. Undocumented Routines Referenced in This Program .......................... 102
8. The Page Control Objects for a Single Page (Illustration) .......................... 104
9. The Programs .......................... 105
   1. page_fault .......................... 105
   2. read_page .......................... 107
   3. find_core .......................... 108
   4. try_to_write_page .......................... 111
   5. write_page .......................... 112
   6. allocate_pd .......................... 113
   7. page_is_zero .......................... 114
   8. get_free_pd_record .......................... 116
   9. post_page .......................... 118
  10. start_rws .......................... 119
  11. rws_abort .......................... 120
  12. rws_done .......................... 121
  13. Small Auxiliary Routines .......................... 123
  14. Typical Paging I/O Routine .......................... 125
## List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Behavior of Anomalies Resulting from Deletion</td>
<td>37</td>
</tr>
<tr>
<td>3.1</td>
<td>Linear/Linear Plot, Exception Ratio vs Memory Extension</td>
<td>48</td>
</tr>
<tr>
<td>3.2</td>
<td>Exception Ratio vs Memory Extension, Logarithmic Exception Ratio Axis</td>
<td>49</td>
</tr>
<tr>
<td>3.3</td>
<td>Lower Region of Figure 3.1, Linear/Linear Plot</td>
<td>51</td>
</tr>
<tr>
<td>3.4</td>
<td>Exception Ratio vs Memory Extension, both axes logarithmic</td>
<td>52</td>
</tr>
<tr>
<td>3.5</td>
<td>MHBPF vs Memory Extension</td>
<td>53</td>
</tr>
<tr>
<td>3.6</td>
<td>Figure 3.1 Corrected for Worst-Case Deletion Error</td>
<td>71</td>
</tr>
<tr>
<td>A.1</td>
<td>The Page Control Objects for a Single Page</td>
<td>104</td>
</tr>
<tr>
<td>C.1</td>
<td>User Load During Experiments</td>
<td>136</td>
</tr>
<tr>
<td>C.2</td>
<td>Percent of System Idle During Experiment</td>
<td>137</td>
</tr>
<tr>
<td>C.3</td>
<td>Percent of System Spent in Page Fault Overhead During Experiment</td>
<td>138</td>
</tr>
</tbody>
</table>
Introduction

1. Brief Statement of the Problem

In this thesis, we describe and report the results of an experiment designed to predict the performance of automatically managed multilevel memory systems for a previously unexplored range of primary memory sizes.

2. Summary of Result

We have developed techniques for predicting memory system performance on an operative computer utility, utilizing an automatically managed multilevel virtual memory. Based upon established theoretical techniques, we have developed techniques to extract the necessary data from a computer utility functioning under a live load. In doing so, we considered problems of dynamic creation and deletion of pages which apparently have not been dealt with previously. The viability of these techniques was demonstrated by performing several measurements.

Using these techniques, we have found that, on the measured system, the rate of accesses to data outside of primary memory decreased drastically as primary memory size is increased above $2 \times 10^8$ bits (6 million 36-bit words, or 24 megabytes). We have found that the mean time between these accesses, as a function of primary memory size was best approximated by a function of at least the second order, and possibly exponential. Previous research on the system under consideration showed a linear function to hold for primary memory size up to $1.3 \times 10^8$ bits (4 million 36-bit words, or 16 megabytes) (81). Although these results do not attempt to characterize Multics, we believe that they are rea-
sonably representative of the observed class of user behavior.

3. Summary of the Work of this Thesis

By means of an experiment on the Multics computer system (B2), running on the Honeywell 645 at M.I.T., we have arrived at measurements of the predicted reference rates to secondary memory for hypothetical extensions of primary memory. These measurements were made on an actual user load, the M.I.T. community, and not any sort of benchmark or test load. From these measurements, models of program behavior in LRU*-managed storage hierarchies can be derived. We suggest here one such model.

The essential technique for deriving these predictions from such measurements is known in the literature (C1,C2) as the "extension problem". It is based upon the properties of a class of memory management algorithms known as "stack algorithms" (M1), which include LRU. Using these properties, we were able to simulate the operation of the LRU algorithm for larger primary memory sizes than the actual one present for the identical user load. The input to this simulation was a history of all references to data outside of primary memory, specifically, on disk, during the period of measurement. It is a property of the stack algorithms that one measurement and simulation can be used to predict secondary memory reference rates for all primary memory sizes.

The work reported in this thesis is significant because it is both the first measurement of this type on a paged, segmented, multiprogrammed computer system which has been reported, and an extension of our range of

*LRU, for Least Recently Used - a memory management policy whereby the least recently used data is moved to slower memory when space is needed in faster memory.
knowledge of the so-called "headway" function which we have described above. Previous measurements of this function (S1) involved other techniques, and only investigated it for primary memory sizes of up to $1.3 \times 10^8$ bits. Our measurements explored regions approaching $4 \times 10^8$ bits. Although there is no inherent limit on the range which could in principle be explored by our techniques, the limitation of our explorations is due only to the noteworthy fact that over a day's running of the experiment, no more than $4 \times 10^8$ bits of information were referenced more than once by the M.I.T. community.

The significance of the actual resulting measurement is twofold: First, it provides an example of typical behavior for the measured system. Second, it suggests more general models of program behavior.

4. Structure of this Thesis

Chapter 1 discusses the concepts of paging and virtual memory. We provide justification for the types of statistics and models we seek and describe how to use them in performance predictions. We discuss previous research in this area, and provide a more detailed statement of the novelty of this thesis.

Chapter 2 describes the experiment. We describe the relevant features of the so-called "stack" algorithms (S1), and the extension problem. We discuss the problems of adapting this type of experiment to the multilevel memory system of Multics. We describe the difficulties in performing this experiment on an operating computer utility, and the solutions we adopt.

Chapter 3 gives the results of the experiment. The results are pre-
presented graphically, and we suggest their interpretation. We analyze these results, and provide a detailed error analysis.

Chapter 4 is a summary of the work done. We suggest future directions for research, and pose some of the questions left unanswered by this thesis.

There are three appendices.

Appendix A is an extremely detailed description of the Multics paging control algorithm, as it was at the time of the experiment. We describe it on several levels, allowing comprehension by the reader on whichever one he chooses. This background is useful for full comprehension of certain design decisions in the planning of the experiment. It is also the first publication of this algorithm at this level of detail (Corbató (C4) provides a less detailed discussion).

Appendix B describes how the actual events of Multics memory management were mapped into the idealized events of theoretical interest to the experiment. We describe the modifications and the interface to the Multics supervisor necessary for this experiment. We assume that the reader has some comprehension of the previous appendix.

Appendix C is a graphical presentation of user load, idle time, and paging overhead on the Multics system on the days of the experiment. These figures were derived from routine metering performed by the administration of the M.I.T. Information Processing Center.
1.1 Memory Performance Prediction as a Goal

As digital computer systems have increased in size and complexity since their inception almost twenty years ago, so have the memory architectures required to support increasingly advanced applications and systems. What is more, progress in memory technology has created a plethora of memory media, ranging over a wide gamut of costs, speeds, and properties. The desire for increased throughput, and in real-time systems, the desire for quick response, create a need for the fastest memory technology available. The fastest media, however, are almost always the most expensive on a cost-per-bit basis. Thus, for a given computer system to achieve or approach desired goals of memory access speed within a given economic constraint, it becomes useful for memory systems consisting of varying amounts of mixed memory technologies to be used in one installation.

Most computers of the past twenty years have used magnetic core as their main, or primary memory. That is to say, the processor was capable of fetching data and instructions only from core memory. Further memory demands were met by the use of tapes, disks, and other bulk media, whose contents could be transferred in or out of selected areas of primary memory by explicit program request. Most of the programs and operating systems designed for this type of architecture allocated these areas for input/output transfers in fixed, specific regions of primary memory. When programs could not fit in their entirety in primary memory, they were divided into independent pieces, or overlays, which were transferred...
in and out of primary memory essentially at their own discretion.

In the last few years, a strategy known as virtual memory has achieved popularity. With this scheme, programs are allowed, effectively, to reference data or instructions in primary memory or on any secondary storage device in an identical manner, creating the impression of a very large, or in some cases, conceptually infinite primary memory. References to secondary memory cause software intervention, signalled by specialized hardware, which results in selected code or data fragments being read into primary memory. Clearly, this implies replacement of some other code or data currently in primary memory, and in order to facilitate this task, such systems divide all primary and secondary storage into equal-sized areas, called blocks, or page frames. Information in the system is divided into pages, which may reside in various page frames at various times. This implementation of virtual memory is thus known as demand paging, as pages are read in on demand, i.e., when referenced. The selection of appropriate pages in primary memory for replacement is a critical issue, and is still a basis for much further study.

A page fault, as the software-assisted fetch of a page not in primary memory is called, represents lost time. The time required to access and transfer the copy of the page on secondary storage is time during which the requesting program may not run. The time that a processor must spend in page fault software, deciding on an appropriate page to replace, is a system overhead, which does not contribute to the progress of users' programs. Multiprogramming, a scheme almost universally used on medium and large scale systems, allows processors to serve one user's program while another's is suspended, say for a page fault. But even here, most
systems limit the degree (number of simultaneously runnable users) of multiprogramming, and page faults can lead to a situation where a processor spends an undesirably large fraction of its time sitting idle, accomplishing no function at all. Furthermore, the primary memory space occupied by all of the faulting program is unusable by any program for the duration of the transfer. Thus, the minimization of page faults in a virtual memory system is extremely desirable. It is an important function of the page-replacement algorithm, as the procedure which selects pages for replacement at page-fault time is known, to attempt to minimize the number of page faults in the foreseeable future. These decisions are usually made with information gleaned from observation of page usage in the immediate past, occasional knowledge of predicted page usage patterns, and some general models of program behavior.

Many page-replacement algorithms have thus been designed for virtual memory systems with the explicit objective of minimizing page faults. These algorithms are subject to mathematical analysis, which is not true of arbitrary user programs. Hence, by careful observation of the storage references made by a program or multiprogrammed collection of programs (although the latter clearly requires some further remarks) we can analyze its interaction with any given page-replacement algorithm running in any given size of primary memory, and ascertain which page faults would or would not have occurred had primary memory been some other size. These techniques are not in general applicable to non-virtual memory systems, for many programs have no idea of how large a memory they are running in, or how to take advantage of it, and thus explicitly-requested data transfers are not affected by changing memory size in any inter-
esting or easily analyzable way.

The ability to determine page fault rates (page faults per unit time) for different memory sizes is a powerful tool in both performance analysis and memory system engineering. Sekino (S2) has shown the explicit dependence of response time and throughput in multiprogrammed systems on the mean headway between page faults (MHBPF). This quantity describes the mean amount of useful work done by user programs between each two page faults. It is most conveniently measured in total references to the virtual memory. If the mean amount of system overhead associated with a page fault is known, as well as a proper characterization of system idle time, we may compute MHBPF from the mean real time between page faults (MTBPF) and the processor reference rate. Hence, predictive techniques to obtain page fault rates for contemplated memory sizes can be used to deduce the system throughput and response time figures which would result. Hence, if one can indeed predict these figures, the economic tradeoffs involved in acquiring improved memory system performance by increasing primary memory size may be evaluated more methodically.

The use of more than one type of secondary memory in a single system results in a situation where the average time to access a data item in any part of the storage system is a function of both the average access time to a data item in each unit and the probability of accessing that unit. In a demand paging system, the probability of accessing each unit is the sum of the probabilities of accessing each page stored on it. If one can associate these probabilities with given pages of such a system, one can create a composite memory system with an optimal average access time within any given cost constraint. Ramamoorthy and Chandy (R1) have
given an algorithm, whereby such a system may be constructed out of any
collection of memory types, whose speed and cost-per-bit characteristics
are known. In any case, it is clear that one should keep the pages with
the highest reference probability on the fastest storage devices. Al-
though the identities of these pages may be determined by experimentation,
observeration, and program analysis, one can view these probabilities and/or
identities as functions of time. Thus, one can devise algorithms which
attempt to maintain pages with given ranges of next-reference probabilities
on appropriate storage devices. It should be fairly apparent that this
problem is identical to that of maintaining pages in primary memory with
the intent of minimizing page faults. This will be discussed more later
on. Thus, the design of an optimal multilevel storage system, as such
configurations are known, can also be analyzed by the techniques of pri-
mary memory paging analysis. Again, the assumption of an appropriate
model of program behavior, both in general and for the particular system
at hand, is of crucial importance.
1.2 Program Reference Patterns and Models

Computer programs being among the most deterministic of all things, any characterization of the data reference patterns of any particular program may be obtained by the simulated running of that program and the observation of whatever characterizations are desired. However, system engineering requires characterizations of programs which are to be run, which, for the most part, have not yet been written. In a given computer system, running under a given operation system, most running programs have many features of their memory usage patterns in common. For instance, in an operating system where an Algol-60 or PL/I-like run-time stack is native to the environment, the pages containing the top of the stack will always have a higher next-reference probability then page representing lower regions. If the supervisor itself is paged, i.e., running in a virtual memory, the same as users' programs, the supervisor has its own reference patterns which will be present in any run of the system. The same is true of compilers, assemblers, system utilities, library routines, and other service programs. Code generated by the same compiler is likely to produce certain common features in its reference patterns, particularly on a local level. Thus, there is great value in observing typical behavior of programs in a large computer system, and trying to formulate some model which is in some sense average or typical.

In a multiprogrammed system, this averaging is done for us in real time. An experimental observation of program behavior in a multiprogrammed computer system, made over some reasonable period of time, say a day, will produce a characterization of typical system behavior, if one indeed believes that such exists. This characterization takes into con-
sideration all of the programs run in that day, and relies on the common features of programs discussed above to have any validity at all. The interval of a day is chosen as reasonable, for that is the cycle time of many forms of human interaction with a computer. People deal with an interactive computer system for several days, doing the same type of work at similar hours in the day.

The particular model of reference behavior that we seek describes next-reference probability of pages in a virtual memory system as a function of position in a certain dynamic ordering, known as a stack, imposed by the page-replacement algorithm. The class of algorithms amenable to this analysis are precisely those which would keep the top \( n \) pages of this ordering in an \( n \)-page primary memory, were it used to manage such. This will be discussed more fully in section 2.1. What is important here is that we can arrive at a function \( p(x) \), where \( p \) is the probability of reference to position \( x \) in this ordering. It is the object of that sub-class of these page-replacement algorithms which are actually useful for memory management to make this function monotonically decreasing. If the algorithm actually succeeds at this, it is clear that then pages which are most likely to be referenced will probably be in the \( n \)-page primary memory, and thus, the page-replacement algorithm has succeeded in minimizing references outside of the \( n \)-page primary memory, or page faults. In the case of multilevel memories, we can pick out whatever positions in the ordering are appropriate, by Ramamoorthy and Chandy's algorithm, and assign them to whatever storage unit is required. C. K. Chow (C3) has also given an algorithm where an optimal multilevel memory system within a cost constraint may be constructed directly from the function \( p(x) \).
1.3 The Experimental Determination of Predicted Headways

If we accept the function \( p(x) \) as a valid characterization of average and typical behavior in a multiprogrammed system, we may predict page fault headways for hypothetical memory extensions from it. Furthermore, the function \( p(x) \) may be measured experimentally. In this section, we show how to approximate and use \( p(x) \) in this way.

\( x \) is the position of a page in the algorithm-imposed ordering we have been discussing. Assume we have constructed the necessary tools to measure \( r(x) \), where \( r(x) \) is the number of times a page in position \( x \) of the ordering was referenced. Assuming pages which were never touched to be in position "infinity" of the ordering, then the relative frequency of touching a page in position \( x \) is

\[
f(x) = \frac{r(x)}{\sum_{t=1}^{\infty} r(t)}
\]  (1)

Here, the numerator is the count of references to position \( x \), and the denominator is the total number of references to all positions. If \( p(x) \) is indeed a valid characterization, \( f(x) \) should approximate \( p(x) \).

We have stated, that for the class of algorithms under consideration, the first \( k \) positions of this ordering at any time contain precisely those pages which would be in a primary memory of size \( k \). Hence, references to pages in the first \( k \) positions of the ordering never cause a page fault in a \( k \)-page primary memory, and references to pages in any position beyond \( k \) always cause page faults. Hence, if a program makes \( H \) references to the virtual memory, the number of page faults it will take in the course of those references is identically the total number of references made
to positions in the ordering beyond the primary memory size. Thus, the program, running in a k-page primary memory, will produce a mean headway

$$MHBPF(k) = \frac{H}{\sum_{t=k+1}^{\infty} r(t)}$$

(2)

This relation holds true for any primary memory size k. If we have an actual system, running in a primary memory of size n, we can predict the MHBPF which would result on this system were memory extended to size E, E being greater than n. We assume that we can measure MHBPF(n) on the existing system, and that a tool for measuring r(t), for t>n, is available. Then the same program which takes H virtual memory references will have a MHBPF in the E page memory of

$$MHBPF(E) = \frac{H}{\sum_{t=E+1}^{\infty} r(t)}$$

(3)

We now divide equation (3) by equation (2), obtaining

$$\frac{MHBPF(E)}{MHBPF(n)} = \frac{\sum_{t=n+1}^{\infty} r(t)}{\sum_{t=E+1}^{\infty} r(t)}$$

(4)

Observe that this equation allows us to predict MHBPF from a measured MHBPF and measured reference counts, a fact which will be used later. We now rewrite equation (1) to read

$$r(t) = f(t) \sum_{u=1}^{\infty} r(u)$$

(5)

letting t be what was x and u be what was t. Substituting (5) in (4),
replacing \( r(t) \), we obtain

\[
\sum_{t = E+1}^{\infty} f(t) \sum_{u = 1}^{\infty} r(u) = \sum_{u = 1}^{\infty} r(u) \sum_{t = E+1}^{\infty} f(t)
\]

\[
\frac{\sum_{t = E+1}^{\infty} f(t)}{\sum_{t = E+1}^{\infty} f(t)}
\]

Multiplying both sides by \( \text{MHBPF}(n) \), we obtain

\[
\frac{\sum_{t = E+1}^{\infty} f(t)}{\sum_{t = E+1}^{\infty} f(t)}
\]

This equation states that mean headway between page faults which would result from a memory extension to \( E \) pages may be computed from the measured mean headway between page faults on the unextended memory, and a factor which is a function only of the program or programs being run and the memory sizes concerned. The work of our thesis is to compute this factor.
1.4 Previous Work in this Area

Since the advent of virtual memory computer systems, the function $M(NPFF(x))$ has been of great interest, being an easily identifiable characterization of memory system performance. Investigators have run many programs in simulation, obtaining this mean headway as a function of memory size experimentally. Almost all of these experiments have been done on machines which attempt to 'compress' a program into a smaller space than that in which it was intended to run. Such systems may typically attempt to fit five or ten programs, each running in a 32 k virtual memory into a core memory of 96 to 150 k. In such instances, the set of pages referenced by each program is small, as is the potential set which it can reference. These sets of pages are usually disjoint, as they represent disjoint virtual memories. Virtual memory in this case is simply a technique to force several programs into a primary memory too small to contain all of them.

Such work has been reported by Belady (B1), Belady and Kuehner (B3), and Fine et al. (F1), among others. A large amount of this work was done on an IBM M44/44X, a 7040 type machine at IBM Research Labs adapted to demand paging. Belady and Kuehner report an expected HBPF for single programs running on this system of the general form $e = a n^2$, $n$ being primary memory size.

Brawn and Gustavson (B4) performed some measurements of typical computational programs running on the same M44/44X. These measurements were significant as they are apparently the first reported measurements of programs specifically written for a virtual memory. They observed the
running time of programs, including page fault overhead, as a function of real memory size. No analytic models were suggested.

Some performance analysis done by Schwartz (S3) on a Burroughs model 6700 is also of interest here. In this system, all data available to a program is referenced as variable-size segments, brought into core on a demand basis. Program code and certain data segments are shared, and the amount of information potentially accessible to a program is extremely large. He reported headway functions of the form $e = \exp(a \cdot n)$, variables the same as above, for missing-segment exceptions as memory size was varied. (These were actual measurements performed on various memory configurations.)

The research which directly led to this thesis was done by Saltzer, and later by Saltzer, Webber, and Snyder(S1). Saltzer measured the MHBPF on the Multics system (B2) at M.I.T., with two different sizes of configured memory. He obtained the result $e = a \cdot n$, which has since been called the 'linear paging model'. Saltzer later reported the results of an experiment designed and conducted by Webber and Snyder, in which the reorderings of the list by which the Multics paging drum is maintained were observed. Using the techniques described in 1.2 above, MHBPF(n) was extrapolated to a memory size of 4000 pages (each Multics page is 1024 words by 36 bits), and was found to be still within experimental error of the linear paging model.
1.5 Novelty of the Work in this Thesis

The work performed in this thesis was originally conceived as an extension to Saltzer and Webber's experiment, elucidating the nature of MBRPP(n) for n greater than 4000 pages. The limitations of the linear model were sought, as was the nature of whatever model held beyond that range.

This series of experiments on the Multics system is unique for several reasons. The data accessible to any program in Multics is potentially the entire storage system, and all data accesses are made via the virtual memory mechanism. This is similar to the Burroughs scheme, but dissimilar to the paged 'compressing' type systems described above. Furthermore, sharing is an extremely important consideration in Multics, as all program code, including the supervisor, is shared. This thesis is also apparently the first reported attempt to deal with dynamically variable virtual memories, i.e., those whose size grows and shrinks on a second-to-second basis. The issues of dynamic page creation and destruction which result from this policy are systematically dealt with by our experiment.

The use of virtual memory seems to be gaining in popularity as large general-purpose information systems become more common. Increased interest in systematic protection schemes has resulted in many new designs for systems having segmented addressing features similar to those found in Multics. Demand paging has achieved considerably more popularity and widespread use than the Burroughs techniques as an implementation for segmentation, and has recently been added by IBM to their extremely popular System/370.
For these reasons, we feel that experiments made on a Multics-like system are relevant to data systems in the near future, and the reference patterns observed may have some features which are in some sense characteristic of programs running in segmented, paged, environments.
2.1 Stack Algorithms and the Extension Problem

The substance of the experiment performed was to reconstruct the entire history of a day's LRU-maintenance of the Multics storage hierarchy, and attempt to predict page-fault headways for hypothetical memory configurations from this history.

The basic strategy of memory simulation used was that proposed by Mattson et al. (M1). This technique, known as stack simulation, relies on the fact that a large number of useful paging algorithms, including LRU, have the property that after any fixed number of addresses in an address trace have been processed by the algorithm, the pages which are left in primary memory are always a subset of what they would have been at the same point in the trace had primary memory been larger. This feature, known as the "inclusion property", thus defines the class of "stack algorithms". From this property, at any given point in the processing of an address trace an ordering can be constructed. The first page in this ordering would be that page which would then be in primary memory were it of single-page capacity, the second would be that page which would also be in primary memory were it of two-page size, the third that which would be added were memory of three-page size, and so forth. The history of the processing of an address trace can be viewed as a series of these orderings, which are known as "stacks", the single page corresponding to unit-size memory normally being considered the "top". As each new reference is processed, the algorithm causes the stack to be reordered, possibly corresponding to page motion for some size memory. The top n
pages on the stack being the pages which would be in a memory of n-page capacity, any motion of a page into the top n-pages implies a physical reading of a page into primary memory. For a demand stack algorithm, this movement can occur only as the result of a page fault. Thus, we may infer the behavior of an n-page primary memory by observing the number of times that reference is made to position n+1 or beyond in such a stack. As we have defined this stack and the class of algorithms processing it as maintaining the first n pages in this stack in an n-page memory, no reference to position n or below can ever cause a page fault. Mattson's technique consists of taking a recorded or proposed address trace, running it through a program which constructs the sequence of stacks just described, and accumulates the total number of references to each position therein. When the processing of the trace begins, the stack is void, corresponding to an empty primary memory. At least until a given page is fetched into primary memory the first time, it will not have been in the stack at all, and its first fetch may be considered to have been made from position "infinity". As the trace progresses, and repeated references to pages are made, we accumulate counts for each position in the stack of how many times a page in that position was moved upward by the algorithm. It can be shown that for a demand stack algorithm, the only condition on which a page may move upward in the stack is that it is that page which has just been referenced. Simply, were this not the case, a page in position n would move into an n-1 page primary memory without having been referenced, and the algorithm would not be a demand paging algorithm. As the completion of the address trace, we can, for any n, sum the reference counts for positions n+1 to the total final length of the stack, plus the
count for position "infinity", and this will be the number of page faults which would have transpired had that address trace been managed by the algorithm used in an n-page primary memory. Note that a single processing of the trace can be used to produce a result which can then be used to analyze any hypothetical memory size.

This technique allows us to ascertain the page fault count for the interval under consideration for any contemplated memory size. By simply dividing the total system headway during the actual trace by this page-fault count, we may thus ascertain the predicted mean time between page faults (MHTPF) for that storage system. Furthermore, we can plot the reference counts at each position, normalized with respect to the total number of reference counts, versus the position number, and obtain a graph which describes what we shall call access frequencies. With this, we can analyze the behavior of multilevel memory systems processing this trace, and obtain an optimal such system within cost constraints as described in Chapter 1. The shape of this graph also tells us much about the relative success of the particular algorithm in managing that particular address trace, without regard to any single memory configuration. We will consider the particular graph in the case of our results in greater detail in the next chapter, and in so doing further consider such graphs in general.

Our experiment sought to learn the shape and nature of this graph at positions corresponding to memory sizes of many thousands of pages. In order to record a reference to position n in a stack as described, there must clearly be n-1 items above it. This implies that at least n distinct items have been referenced by the time a reference to the n\textsuperscript{th} posi-
tion occurs. It can be seen that the extent of the address trace required
to produce meaningful statistics at the ten-thousandth page position would
require a prodigious address trace. At this point advantage may be taken
of another remarkable property of stack algorithms. It is possible to con-
struct the portion of the stack from position n+1 to the end without a full
address trace -- we use information extracted from a running algorithm
managing an n-page primary memory at times when page faults occur. This
is known as the "extension problem" (C1,C2). The technique is as follows:
we maintain the stack (the "extension stack") for positions n+1 and be-
yond. When a page fault occurs, we know that the page faulted on cannot
be in the first n positions of the stack -- if so, it would not have been
faulted on. We locate the page in the extension stack; if not there, we
may consider it as having been at position "infinity". The counter cor-
responding to the position from which the page was fetched is incremented.
We remove the page in question from the extension stack: it is now in the
top position of the real stack, which we are not maintaining. We now use
whatever information is necessary, from that normally obtainable to the
running algorithm, plus that we are maintaining, to reorder the extension
stack according to the policy of the running algorithm. This reordering
will usually include placing some page removed from primary memory by
the running algorithm at some point in the extension stack. In the case
where the replacement algorithm is LRU, the page removed from primary
memory is placed on top of the extension stack, and all pages previously
in the extension stack move down one location. Note that pages which
were below the fetched page in the extension stack stay in place during
the entire transaction.
The advantages of using a trace of a running algorithm in a large system measured over an extended period of time, as opposed to a trace obtained by simulation of a given program over a necessarily much shorter period of time are straightforward. We are interested in system performance on an hour-to-hour, not second-to-second, basis, and day-long measurements of a live system correspond to both the time scale and load mix of interest. As long as the accuracy of the measurement can be maintained, this day-long extension measurement is much more useful than the simulated running of a program.

As a demonstration of the power of this extension technique, we may consider the Multics system: 400,000 references to the virtual memory occur every second. Approximately 100 page faults occur each second. Recording 2 data items for each page fault, we have reduced the amount of data which must be recorded by a factor of two thousand.

It should be clear that the results of this experiment, although simulating hypothetical memory system performance, do not represent simulated results. The measurements made correspond to an uncontrolled user population during normal working days, using arbitrary programs under 6-way multiprogramming. The results thus show how a hypothetical memory system would have behaved under this real user load.
2.2 The Extension Problem and Multics

The Multics system has a physical memory consisting of 256K (1 K = 1024 36-bit words = 3.7 x 10^4 bits) to 384 K words (1.2 x 10^7 bits) of core, a 2000 to 4000 K word (1.5 x 10^8 bits) drum, and approximately 90,000 K words (3.3 x 10^9 bits) of disk, both moving and fixed head. The variabilities stated above are dependent upon the time of day and the user load, governed by administrative policy. The entire storage system is divided into 1024-word pages, and is managed by the demand paging mechanism (with the exception of several thousand words of non-pageable code and data, such as the code for the paging mechanism itself, which must be non-pageable in any case, and are thus not really of interest in memory performance prediction). The algorithm used to manage replacement of pages in the core memory is essentially LRU. The variation from LRU is explained in detail in Appendix A. Essentially, within the constraints of operating system overhead and the precision of measurement of recency of use provided by the hardware, it tries to implement LRU as closely as possible. Also, a non-demand prepage/postpurge policy was in effect during these measurements, which caused some pages to move in and out of core outside of the control of the LRU algorithm.

The 4000-page paging drum was at this time being used in a mode which attempted to overcome rotational latency by making multiple copies (S4), in this case two, and hence was of 2000 page capacity during these experiments. Since January, 1972, the drum has been used as part of a hierarchically managed storage system, as a buffer between core and the disk storage subsystem. In such systems, one attempts to keep pages with the highest access frequencies on the fastest devices, in order to
minimize the system's mean access time. In an automatically managed system, the identity of those pages is constantly changing. As the stack access frequency graph discussed above can be used to associate access frequency with stack position, page replacement algorithms identical to those used to manage primary memory are frequently used to manage other devices in a hierarchical memory system to achieve this end. In the MULTICS system, another near-LRU algorithm is used to manage the drum, which is described as well in the Appendix. The drum management algorithm attempts to maintain copies of the top 2000 pages of the theoretical stack corresponding to the LRU algorithm on the drum. The model of program behavior implied by the LRU algorithm, and verified by the results of this experiment, implies that these pages are the most likely to be referenced, and at the time they are on the drum, thus have the highest access frequencies.

As currently implemented, a page which has been faulted on, and is not on the drum, is read into core from the disk. It will not be written to the drum until the core management algorithm decides to oust it from core. This implies that the pages corresponding to that portion of the LRU stack representing core are not completely a subset of those on the drum. Hence the drum will contain pages representing a 2000-page contiguous portion of the stack, whose topmost extreme is anywhere between the top of the stack and the size of core below it. Of the 256 to 384K, about 100K is not used for paging, leaving 150 to 280K for paging, thus this variability represents about 7 to 15% of the size of the drum.

The stack-reordering procedure of the LRU algorithm is one of the simplest possible: the referenced page moves to the top position of the
stack, as is necessary in any demand stack algorithm. The pages between
the top page and the old position of the referenced page all move down one
position. Thus, to use the extension technique described above for the
LRU algorithm, the only reordering information one need record is the
identity of the page thrown out of position \( p \) (\( p \) being the size of the non-
extended memory, in pages), which will then occupy position \( p+1 \), at each
page fault, in addition to the identity of the page faulted on. The
"pushed" page becomes the top of the extension stack, the page previously
there becomes \#2, etcetera, all the way down to the former position of the
faulted-on page. This is what we have done with the Multics core-drum
combination, considering it as a 2000+X page buffer, where \( X \) is some frac-
tion of the size of core, itself at most fifteen percent of the drum, to
account for the top-of-drum variability described. As positions \( p+1 \) and
on, in our case, correspond to the disk subsystems, we need only record
disk reads instead of page faults. (It is instructive to note that within
the entire operation of a Multics system, not a single direct-access I/O
transfer is done outside the paging mechanism, pre-paging being included
in this consideration.) As disk reads are two to five per second, we
have thus reduced our data-gathering chore by at least 95%. The experi-
ment of Saltzer, Webber, and Snyder, which was similar in intent to this
experiment, but more limited in scope, has already produced results (S1)
for primary memory sizes up to the maximum size of the drum. For this
reason, we did not consider it worthwhile to attempt to gather data for
that portion of the stack corresponding to regions in the drum. Hence,
the application of the extension technique to this core-drum combination
was adequate. All else that was needed was the recording of information
provided by the drum management algorithm as to the identity of pages
thrown off the drum. It can be seen that these are the pages thrown out
of the core-drum combination, given that no page is ever thrown out of
core without having been written to the drum.

In order to determine the validity of this technique, the necessary
programs were written and tested on a stand-alone Multics machine. This
machine had 131,072 words of core and a 256-page drum. Hence, most of the
range of the regular Multics drum was in the extension region of this ex-
periment. The mean headway curve resulting was very well approximated by
a straight line, suggesting the linear paging model. This provided a
good deal of confidence in both the technique and the software.

Hence, we see two types of motion between core-drum and disk. The
reading of a page constitutes motion from disk into core-drum. The
writing of a page, however, does not constitute outward motion. In gene-
ral, writing is performed only when a copy of a page on disk is different
from a drum or core copy. The outward motion corresponding to a read is
really the claiming of the core or drum frame previously occupied by the
page of interest. We call this phenomenon an "ouster".

Unfortunately, a problem arises with even this simple model. Certain
pages of the storage system, 3 in all, corresponding to the system's top-
level directory, are special-cased by the paging and drum-management al-
gorithms such that they may never go on the drum. This is due to certain
integrity issues involving the reliability of the drum and the extreme
difficulty in reconstructing the contents of this directory. Hence,
these pages are never written to the drum, and leave the "core" portion
of the increasingly less theoretical LRU stack directly for the disk por-
tion. (In this case, writing never takes place unless the concerned page has actually been modified (see Appendix A)). More unfortunately, these are among the most popular pages on the disk, as by the dictates of the LRU algorithm, they should have by all means been on the drum. Thus, we must check for pages being ousted directly from core to disk, and we thus have two varieties (core and drum) of ousting to be recorded. The interpretation of the data resulting from movement of these pages will be deferred until Chapter 3.

Thus, we need record upward stack movement into the core-drum combination, meaning disk reads, and downward movement, meaning oustings. Another event of interest is the creation and deletion of pages. In the current implementation of Multics, logical pages are created out of the void when a never before referenced page is referenced. By definition, all such pages contain zeros, and hence never involve disk reading. Furthermore, these page faults will occur regardless of what size primary memory is, and are thus not of interest in memory performance prediction. This last statement is somewhat subject to current design and user behavior. Were there a tremendous amount of fast, cheap primary memory, it is altogether possible that users would rarely delete programs or data, but simply rewrite or modify them, thus making page creation a much rarer event. We choose to ignore this possibility.

In the following discussion, "n" represents the size of "primary memory", in pages, in terms of the extension problem. In terms of the specific experiment on Multics, n is the size of the core-drum subsystem in pages. As was explained earlier, this is the size of the drum (2000 pages) plus a fraction of the size of core.
Page faults which cause creation of pages involve neither disk traffic, idle time, nor multiprogramming, and are thus not of interest in MTBF calculations. Since all new pages are created in this way, we find them naturally falling past position \( n \) of the complete LRU stack, into the extension stack after sufficiently long disuse. Page deletion, on the other hand, can occur at any time in the life of a page. If a page is destroyed so soon after its creation that it has never passed position \( n \) in the stack, we are oblivious to its entire existence. If, however, it is destroyed at such time that it is beyond position \( n \), its destruction must be accompanied by its excision from the extension stack. When a page is destroyed in core or on the drum, the next page to be faulted on replaces it without any page being pushed down the LRU stack. However, the position in the stack of the destroyed page is assumed by the page directly under it. The page fault following a page destruction creates only upward stack motion -- nothing is pushed down.

Consider, in our theoretical \( n \)-page primary memory system, a page in position \( n \) of the LRU stack. This page is now destroyed. A page in position \( n+m \) is now faulted on. In an actual memory system, this page will now be read into primary memory without any page being replaced, the destroyed page having created an empty page frame, but the newly faulted-on page will be at the top of the LRU stack. The formerly first to \( n \)-1st pages now become the second to \( n \)th pages in the stack. The \( n \)-1st (first page not in core) to \( n+m \)-1st pages retain their original stack position. (See figure 2.1). The \( n+m \)'th position in the new stack is in a situation akin to that of the \( n \)th position after the deletion of the page there: the page in the \( n+m \)-1st position cannot come up to fill the void --
### Top of Stack

<table>
<thead>
<tr>
<th>Position</th>
<th>LRU stack before deletion</th>
<th>After deletion of page P6</th>
<th>After faulting on page P11 after deletion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>P1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>P2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>P3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n-2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n-1</td>
<td>P5</td>
<td>P5</td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>P6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>n+1</td>
<td>P7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>n+2</td>
<td>P8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>n+m-1</td>
<td>P10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>n+m</td>
<td>P11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>n+m+1</td>
<td>P12</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 2.1 -- Behavior of anomalies resulting from deletion.
that would not be demand paging. Until some reference is made to a further position, say \( n_{m+k} \), this will be the case. At the time the \( n_{m+k} \) reference is made, position \( n_{m+k} \) now becomes anomalous. Hence, we create an anomaly when a page is deleted, which propagates down the stack as any reference is made beyond it.

The strategy that we have chosen to deal with this, in the simulation, is simply to excise a page from the stack when it is deleted. Thus, any reference to a position beyond the excision will be tallied as a reference to position \( x \) instead of \( x+1 \). Note, however, that the position of the excision has then moved to \( x \). All references in front of excisions are tallied correctly. The analysis of the inaccuracies resulting from this treatment is quite involved, and is covered in detail in section 3.4.3.

Thus, the data items which must be recorded in a trace are those representing 1) reading of pages into memory, by demand or prepaging, from disk, 2) claiming of pages by ousting pages from drum to disk or from core to disk, and 3) the deletion of pages from the storage system. Of these events, types 1 and 3 represent excision of a page from the extension stack, while type 2 represents the pushing of a page on to the top of the extension stack. Events (1) also cause the noting of the stack position of the page read, and the incrementing of a counter corresponding to that position. There are actually some other events which must be recorded in the case of the Multics system, but these are due to the particular implementation of the core and drum management algorithms, and are discussed in Appendix B.

The handling of page reads of pages which cannot be found in the stack, i.e., their first reference, requires some thought as to inter-
pretation. Were this experiment run for a sufficiently long time, the appearance of such pages would cease. Pages which are created come down on the top of the stack, and any existent page which has not been referenced since the experiment began enters the core-drum extension-stack combination once and never leaves, until it is destroyed. These first references, as discussed, are counted in the "infinity" position of the stack. These fetches of pages not in the stack accounted for roughly a tenth (7881/74530) of all disk page fetches. These references do not affect the relative number of fetches to any two extension stack positions, as they would not be in a core-drum memory of any size until the first time they were referenced. Thus, when one considers disk accesses, one should consider these reads to be disk references in a core-drum system of any size. However, the longer the experiment runs, the fewer will these references become. Thus, since we are interested in steady-state behavior, we have chosen to consider these reads a start-up transient, and not count them in any calculation. They tell only of the length of the experiment, not of what is being measured.
2.3 Performing an Experiment on Multics

Having developed the theoretical bases of the extension problem, and adapted it to Multics, the next step was to proceed to construct the necessary software to develop the extension address trace, collect it, and perform the LRU stack simulation with it.

A privileged-access facility was set up in the Multics hardcore supervisor specifically for this experiment. When enabled, a trace of all of the events mentioned above was accumulated in a circular 1008-word buffer. Each trace item included the physical device address of some page being read, ousted from core-drum, or deleted, information as to which of these events is represented, and a flag indicating, for statistical purposes, whether or not it was one of the previously mentioned pages which are not allowed to go on the drum. Also recorded was information allowing the program which inspects this buffer to synchronize itself with it correctly. A program was developed which inspected this buffer regularly--from the Multics standpoint, a privileged operation. This program assembled the buffer images into a continuous trace, which could be as long as necessary, suitable for further, repeated processing.

This strategy was decided upon because of the extensive time required to search an LRU stack for a given page, and the large amount of space required to store this stack. This ruled out the possibility of having a special-purpose module of the Multics supervisor perform the experiment in real time. The performance degradation necessitated by the time required to search and the space, which would have had to been non-pageable, to store the LRU stack would have been wholly unacceptable. Furthermore, the accumulating of the trace data for further processing allows many pro-
grams and versions of programs to be run on this data, increasing both its usefulness and the accuracy of the results obtained from it.

One disadvantage of the data collection strategy described is the possibility of the data collecting program losing synchronism with the circular trace buffer, i.e., data being overwritten by new data before it has been duly noted. This situation can come about when the data collecting program has made a decision about how often the buffer should be sampled, and an intense unexpected burst of activity causes the buffer to be written into significantly faster than before. The data-gathering program samples the buffer again, and notices that data has been lost, but anticipating further loss reschedules itself. Another way that data can be lost from buffer mis-synchronization is the data-gathering process falling behind in the multiprogramming queue due to Multics scheduling policies and heavy user load. The implementation of the data-gathering program tried to compensate for this by being written as a multiprocess program, i.e., a program running in a coordinated way in many processes at once. Not only did this give it a scheduling advantage, but increased the reliability of the data-gathering operation as a whole.

Unfortunately, data losses of the types described were common, especially in initial, developmental runs of this software. The greatest losses would typically occur at midnight, when a large number of user programs scheduled to run then would, creating heavy paging activity referencing pages neither in drum nor core, and only one or two processes would be supporting the data-gathering operation. The extents and analysis of these losses are considered in the next chapter.

A danger of running a large complex data-gathering system in many pro-
cesses is that of creating a great deal of activity which would bias the result of the measurements by measuring itself. The sharing features of the Multics system helped counterbalance this effect: all of the databases and procedures of the data-gathering system were fully shared, having only one copy. Only the per-process work areas were not shared. The actual data-gathering system, in order to handle the control of multiple processes, possibly multiple terminals, and dynamic scheduling was, in fact, quite complex, requiring six separate procedures. The shared databases and procedures totalled ten pages. Approximately two pages of work area per process were needed.

The data gathered was stored in data segments in the Multics virtual memory. The stack simulation was subsequently performed, using this data as the extension address trace, exactly as described above. The procedures which performed this reduction ran in an unrestricted Multics environment, and hence had practically no restriction on time or space. The LRU stack was represented as a list, in which each node represented a stack position. A push of a page onto the top of the stack required the allocation of a new node, and the redefinition of this node as the top of the stack. This node was then made to point to the former stack top. The excision of a page from the stack required locating the node corresponding to this page (each node contained a physical page address), the reallocation of this node, and the reconnecting of the list around it. For trace data representing disk reads, however, it was necessary to ascertain the position in the list of the relevant page. This required a search of the entire list. In order to reduce the work of discovering that a page was not in the list at all, a bit table was constructed, describing, for each possible physical
disk address, whether or not it was in the list at all. This saved the necessity of walking the entire list.

The above list-maintenance algorithm spends a great deal of time searching the list to determine the position of pages in it. Several algorithms were considered to avoid the seemingly crude strategy of linear search, but most of these algorithms caused the list to grow increasingly disorganized, requiring periodic time consuming re-organizations, or required large amounts of data movement, a poor approach in a paged system. Because of the availability of a stand-alone machine which could easily provide the computer time necessary to perform this processing, the development of a better list-maintenance algorithm was not pursued further.

The result of the stack simulation was a table, describing for each position in the extension stack, how many times a page in that position had been referenced. The sum of all of these counts, plus those at position "infinity", represented the total number of all page fetches from disk during the period of the measurement. Although a graphical display of this information is of some interest, the calculation of MHBPF was the immediate objective. Thus, a table was created displaying, versus extension stack position, the total number of fetches observed divided by the sum of the counts for all of the positions further down the stack. For a given position N, the interpretation of this number, x, is as follows: had memory (core/drum) been extended N pages above its actual size, we would make one disk reference under that circumstance for every x references we make now. We thus refer to x as 'references per exception'. Note that we have not included the "infinity" fetches in the 'total reference' count in the actual results shown here, for the reasons dis-
cussed in section 2.2.

One may choose to interpret \( x \) as the "relative increase in mean headway", which is to say, the factor by which mean headway will increase over its current value if the extension of \( N \) pages is made to core-drum. For instance, if "references per exception" were 5 at \( N = 4000 \) pages, the interpretation would be: If we added another 4000 pages to the drum, we would fault to the disk one-fifth as often as we do now. This constitutes, in one sense, the raw data result of the experiment. Now,

\[
\frac{\text{References observed}}{\text{References beyond position } N} = \frac{\text{Mean time between measured references}}{\text{Time duration of experiment}}
\]

\[
= \frac{\text{References observed}}{\text{References beyond position } N} \times \frac{\text{Time duration of experiment}}{\text{References observed}}
\]

\[
= \frac{\text{Time duration of experiment}}{\text{References beyond position } N}
\]

\[
= \text{Mean time between references beyond position } N
\]

\[
= \text{Expected mean time between references were memory extended by } N
\]

Multiplying this number by the measured system headway in virtual memory references during the experiment, and dividing by the time duration of the experiment, we obtain the expected mean headway between page faults were memory extended by \( N \).

We have displayed both references per exception, versus memory extension size, and predicted inter-reference headway, as a function of memory size.

Note that in all of this discussion, mean time computed from an ex-
periment taking many hours must be taken quite literally. The averaging effect over a day of usage varying between a heavy user load and solely the data-gathering program* produces a result which is really applicable to neither, but only a theoretical load somewhere in between. For this reason, we feel that 'references per exception' is a more useful interpretation of the results of this experiment than 'mean time between exceptions'. Attempting to tune a system to the theoretical point described by such measurements will not help the system when it needs the most help.

The reference counts and references per exception were subsequently displayed in printed tabular form, and the references per exception versus stack position plotted on a Stromberg-Carlson 4020 Microfilm Recorder. Some of these results are reproduced and discussed in detail in the next chapter.

*Some more precise descriptions of the exact user load during the experiment are provided in Appendix C.
permanent copy of many papers must be taken with the publication. The following
effect over a gap of several years between a paper near the end and start
The design of the computer
The green-eating program promotes a kernel which is ready to reply to
for titles to neither contain only a specific list nor summaries to perform.

1.1 The Environment of the Computer

- Reason we feel that 'preference' is a more useful
- The experiments was conducted using a certain scheme. The early runs were
- Found heavily from documentation manual. In these cases, we have
- Behemoth into Isambard are to meet a need of this purpose.
- Choices in display and equipment were that a certain type of computer was
- The rest of the reader can use the command to reply in this regard. Some computers, such as Isambard.

volumes are given as an example. The list of the earliest computer is 
traffic manual. In and dan D'H were completed before May 7, 1973.

- As the list in the earlier text excitation and display is only to be available 
but computer need to be used if they want. The computer need to be used

except in the case of the earlier computer.

10:00 All and all for 6:00 AM. For dan 21, there should be about 20 users,
but range between 20 and 30 between 6:00 AM and 9:00 AM. The usage
was being used in a single page, one half of the capacity for all of
both experiments. For dan 21, the processor and the paper's pages of
car are on line for the entire experiment. The processor is on line for
most of the 21, except for the period immediately to March 22. It uses
one processor and 200 pages of computer paper only. The rest and system
and system page faults received for both experiments which are given ac-
accurately in Appendix C.
3.2 The Results of the Experiment

The form that we have chosen to display is that of an "exception ratio", or \( \text{MHBPF}(E)/\text{MHBPF}(n) \), where \( n \) is the adjusted 'primary' memory size (core-drum) as explained in section 2.2, and \( E \) is a hypothetical memory, both in pages. This exception ratio is the quantity expressed by equation 1.6. We plot this ratio versus primary memory extension in figure 3.1. We express the abscissa of our graph as 'memory extension', which is the hypothetical increase of core-drum instead of absolute memory size because of the variability of the size of core-drum as discussed in Chapter 2. The size of core-drum is not the sum of sizes of core and drum, because of duplications, created pages in core which have not been copied out to drum, and possibly even different configured sizes of core. The extension size to core-drum is meaningful however, because the data and results derived from the measured data represent the behavior of a hypothetical extension of the given size, oblivious to all of the above considerations. If a figure for the size of core-drum is needed, 2100 pages is reasonable. The shape of this graph suggests an exponential behavior. Thus, we next plot this ratio on a logarithmic vertical axis, to better view this behavior. This is figure 3.2. The plot almost traverses the graph diagonally, suggesting the straight line which would correspond to an exponential. We have drawn a straight-line approximation, which corresponds to

\[
\text{MHBPF}(E)/\text{MHBPF}(n) = 3.42e^{(E-n)/(7.00 \times 10^7 \text{ bits})}
\]

(1)

The surprising closeness of the dtm 21 and dtm 23 plots gives some confidence in this result. A similarity to the unpublished Burroughs re-
Figure 3.1  Linear/Linear Plot, Exception Ratio vs Memory Extension
Figure 3.2 Exception Ratio vs Memory Extension
Logarithmic Exception Ratio Axis

Exception Ratio

$dtm_{21}$

$dtm_{23}$

Exponential Approximation
Exception Ratio =

$3.42e^{(E-n)/7} \times 10^7$ bits

Memory size above the Drum, in pages

1000 2000 3000 4000 5000 6000 7000 8000 9000
result (S3) mentioned in Chapter 1 may be noted, subject to the limitations discussed there. This similarity goes only as far as that Schwartz noticed an exponential mean-headway function for missing data in primary memory as memory size was increased.

It can be seen that the low regions, i.e., below 2000 pages above the drum, fall short of the exponential approximation suggested. Thus, we provide figure 3.3, in which we display the low region of figure 3.1. This plot shows a decidedly less than exponential behavior. From one viewpoint, this is comforting, as the experiment of Saltzer, Webber, and Snyder (S1) measured this function in this range, and obtained a linear function. However, our plot seems to grow considerably faster than linearly in this region. This can be seen as a noticeably different change in behavior.

An attempt to resolve these contradictions is provided by figure 3.4, in which both memory extension and exception ratio are plotted logarithmically. It is seen that the higher regions of this plot approach quadratic slope, and the increasing slope lends more confidence to the exponential suggested above. However, no uniform quadratic curve suggests itself.

The most that can be said is that MHRPF(E)/MHRPF(n) is a function of at least second order, as E-n exceeds 3000 pages.

We also provide figure 3.5, in which we plot MHRPF(E) directly as a function of memory extension, by multiplying the ordinate of figure 3.1 by the measured MHRPF on both measurements. Note that a given exception ratio does not correspond to the same MHRPF on both measurements, as different mean headways were observed. This is due to the variability of the system
Figure 3.4 Exception Ratio vs Memory Extension, both axes logarithmic
Figure 3.5 MIBPF vs Memory Extension

- MIBPF
- Virtual Memory References
- dtm
- Memory Size above Drum, in pages

Values:
- $6 \times 10^7$
- $5 \times 10^7$
- $4 \times 10^7$
- $3 \times 10^7$
- $2 \times 10^7$
- $1 \times 10^7$
load on these days. The difference, however, is not very significant.
3.3 Reference Probability Models

The characterization of the reference process of a program or group of programs as an M/M/1 queue under reference probability distribution is a system-independent characterization of program behavior. We proceed to derive \( p(x) \) from \( \mathbb{H}(n) \) from equation (1).

Let \( K \) be \( \mathbb{H}(n) \), for the notation used in (1) and (2), let us represent the question \( \mathbb{H}(n) = \sum_{x=0}^{n} \frac{1}{x!} \lambda^x e^{-\lambda} \).

\[
(\sum_{x=0}^{n} \frac{1}{x!} \lambda^x e^{-\lambda}) = \sum_{x=0}^{n} \frac{1}{x!} \lambda^x e^{-\lambda} = \mathbb{H}(n)
\]

We write \( p(x) \) instead of \( \mathbb{H}(n) \) in this discussion as we are assuming a sufficiently small interval between \( n \) and \( n+1 \) to approximate \( p(x) \). Let us use a generating function for \( p(x) \). We thus have

\[
\mathbb{H}(n) = \sum_{x=0}^{n} \frac{1}{x!} \lambda^x e^{-\lambda} \]

or, rewriting,

\[
(n-x)\mathbb{H}(n) = \sum_{x=0}^{n} \frac{1}{x!} \lambda^x e^{-\lambda} \]

We are interested in finding the probability of reference to position \( x \), which must be beyond position \( n \). This leads us to only considering references to positions beyond \( n \). We are only considering references to a position beyond a certain position \( n \).

\[
P(x) = \frac{\frac{1}{x!} \lambda^x e^{-\lambda}}{\sum_{x=0}^{n} \frac{1}{x!} \lambda^x e^{-\lambda}}
\]
Now, by definition of $P(x)$:

The characteristic of the reference position is

$$P(x) = [x]$$

or, if we are using a different notation for the characteristic,

$$P(x) = \pi(x)$$

We have already shown that the characteristics are

$$\pi(x)$$

We can now define the characteristic as

$$\pi(x) = \pi(x)$$

Similarly, if we have

$$\pi(x) = \pi(x)$$

The characteristic of the reference position in

$$p(x) = \pi(x)$$

If we are using a different notation for the characteristic,

$$p(x) = \pi(x)$$

We can now define the characteristic as

$$p(x) = \pi(x)$$

$$p(x) = \pi(x)$$

and

$$p(x) = \pi(x)$$

Similarly, if we have

$$p(x) = \pi(x)$$

The characteristic of the reference position in

$$p(x) = \pi(x)$$

If we are using a different notation for the characteristic,

$$p(x) = \pi(x)$$

We can now define the characteristic as

$$p(x) = \pi(x)$$

$$p(x) = \pi(x)$$

and

$$p(x) = \pi(x)$$

Similarly, if we have

$$p(x) = \pi(x)$$

The characteristic of the reference position in

$$p(x) = \pi(x)$$

If we are using a different notation for the characteristic,

$$p(x) = \pi(x)$$

We can now define the characteristic as

$$p(x) = \pi(x)$$

$$p(x) = \pi(x)$$

and

$$p(x) = \pi(x)$$
the polynomial headway function \( \text{MHP}(x) = ax^k \) giving in general

\[
p_k(x) = \frac{kn}{x^{k+1}}
\]  

which subsumes (8), (10), and (11). The exponential model (9) is the only one of these probability distributions which is characterized by an independent parameter, \( \gamma \). Letting \( \lambda = 1/\gamma \), we rewrite (9) as

\[
p(x) = \frac{1}{\lambda} e^{-(x-n)/\lambda}
\]  

\( \lambda \) has the dimensions of pages. It in some sense characterizes a 'radius of locality of reference' of the programs running. It is the mean fetch depth into the extension stack.
3.4 Accuracy of the Reported Results

The question of accuracy of the results of a supposedly deterministic simulation seems at first to be unnecessary. However, this simulation was based upon a measurement. Thus, the techniques used to interface this experiment to the Multics system (see Appendix B) became a source of inaccuracy. Furthermore, the behavior of anomalous pages (the so-called "global transparent paging device" pages) caused significant deviation from the assumed LRU model. The deletion of pages in LRU list created problems, as an inordinate amount of effort would have been required to handle these correctly (see Chapter 2).

Thus, we will consider three sources of inaccuracy: lost data, global transparent paging device pages, and list deletions.
3.4.1 The Effect of Lost Data

The loss of data was due to failure to retrieve it from the Multics supervisor before it was overwritten. This was a consequence of the circular buffer strategy chosen to solve the problem of real-time storage of this data. These strategies were discussed in detail in section 2.3.

The effect of these losses are twofold: some counters for stack positions were not incremented for lost data, and the ordering of the stack was affected by this lost data. We consider these problems separately.
3.4.1.1 Lost Counter Accuracy

In order to deal with either of these problems, we assume that lost data has no correlation to page reference patterns. We thus deduce that it shares the same distribution over stack position as the successfully accumulated data, and the shape of the resulting histogram is not severely affected by this loss. For the measurement "dtm 23", the most successful and accurate of those made, 435 trace items were lost out of a total of about 200,000 successfully recorded items. This represents a total inaccuracy in counting of less than one quarter of a percent. For the slightly less accurate "dtm 21", 1200 items were lost at various times. Measured against the 150,000 items successfully collected here, this is still less than one percent.
3.4.1.2 Stack Shifting Inaccuracies

This type of inaccuracy, resulting from inaccurate reconstruction of the LRU extension stack, is considerably more subtle, and damaging in its effect. Failure to notice certain movement into and out of the extension stack causes the stack to stray progressively farther from a realistic reconstruction. Items remain in the simulated extension stack which were in fact removed by lost data, and items which should have been pushed on its top are not so pushed. The result of items not being removed correctly is twofold: first, the item will appear twice in the stack when it is pushed out legitimately onto the top of the extension stack in the future, and items further down the stack than the false appearance will have their stack position incorrectly recorded. The double appearance is only a problem because of the latter effect. The stack-managing algorithm of the simulation program used a bit table to record the known presence of every storage system page in the extension stack. Thus, the legitimate pushing (the second time) of such a page has no effect, and the later fetching of that page from the extension stack fetches the correct instance, and the bit table indicates the page as no longer being in the stack.

The result of items not being pushed because of failure to record their pushing is similar. Their absence at the top of the extension stack causes all items below them, which is the entire stack, to have their positions incorrectly recorded. These items appear one position higher than they should be, for each missing item. Thus, until the missing item is later requested from the stack, by virtue of a recorded fetch, all items which were on the stack before the failure to place the missing item will
have their positions incorrectly tallied. Also, a later reference to the unpushed page will be recorded as a transient, not-in-stack page fetch, as discussed in section 2.2. The effect of not accounting this fetch to any stack position was already discussed.

Note that the effect of pushing and then fetching any single page has no effect on the extension stack orderings before the pushing and after the fetching. That is to say, if the pushing and fetching were not recorded at all, only the stack orderings between the two would be incorrect. Thus, if at any one time a data retrieval from the hardcore supervisor notices that X data items were lost, all such push-fetch pairs within the X lost data items have no stack-reordering inaccuracy associated with them, and only the lost-counter inaccuracy occurs. A reference-push pair, on the other hand, causes both types of inaccuracy. Although it seems evident, by locality of reference, that any string of contiguous lost data items must contain a large number of push-fetch pairs, i.e., a page recently pushed out of core-drum is one of the most likely to be fetched back in soon, a more careful mathematical analysis shows this to be false. Based upon parameters derived from the data accurately recorded in "dtm 23", 100 page fetches within a string of contiguous lost data items will statistically include only 4 fetches of pages pushed within the lost data items.

We must thus assume the worst case, that every lost data item was in fact a push or a fetch not properly matched within the lost data. Hence, for the 435 lost trace items in "dtm 23", the effect of losing this data could not have been worse than the pushing of 435 never-referenced-again pages on the top of the extension stack, or the excision of 435 random points from the stack. In the first case, the result is that later
fetches are accounted to lower-numbered positions than they should have been, and in the second case, some later fetches are accounted to positions of higher number than they should have been. In either case, the total effect is that of an uncertainty of either $+435$ or $-435$ on the stack position axis of any derived graph. In reality, the lost data must contain an almost equal number of pushes and fetches. (By conservation of pages, the difference must be exactly the difference between pages created and destroyed in core-drum.) As a result, a typical misplaced page in the extension stack will suffer an average displacement of $435/2$, due to the lost fetches. Hence, the total uncertainty in the stack position axis of any derived graph is not greater than plus or minus one-half the number of lost data items. For "dtm 23", this is 218 positions. Most of our graphs are plotted to a resolution of 500 stack positions. Compared to the 8000 or so positions of interest, this inaccuracy is not very significant.

Summarizing, the effect of lost trace data is seen both as lost accuracy in counting, and uncertainty in the stack-position axis of derived graphs. Both uncertainties are proportional to the amount of lost data.
3.4.2 Global Transparent Paging Device Inaccuracies

This type of inaccuracy results from the special handling of the system's top level directory pages, three in all. These pages are ousted from the core-drum combination prematurely, as dictated by a system reliability policy which attempted to insure the integrity of these pages by keeping them off of the drum. As a result, they were in fact used more often than the pages legitimately at the top levels of the extension stack, and thus, had no right to be in that stack at all. That is to say, they would have been on the drum at almost all times had they not been so special-cased.

The anomalous effect of these pages had been seen early in the work of this thesis. An experiment designed to discover the significance of this effect revealed that on some days, between one-half and one-fifth of all Multics disk traffic was a result of these special-cased pages.

Thus, our experiment was modified to note in its trace data when such pages were being fetched or ousted from core-drum. The mechanism by which this information was transmitted was somewhat ad hoc, and is not shown in Appendix A.

The predominant inaccuracy caused by these pages is a distortion of the very low end of the $f(x)$ and $r(x)$ curves (see section 1.3). The stack-reordering inaccuracy created by these pages cannot be more than plus or minus three positions (as there are only three of these pages) at any point in time or stack, and is thus totally insignificant. As these pages rightfully belong on the drum, they are usually fetched very soon after they are ousted, and thus, never migrate very far down the stack.

Thus, many reference counts at low-numbered stack positions are attributable to these pages. If the core-drum combination were extended by
any finite amount, and managed as currently done (i.e., at the time of the experiment), the anomalous references would still appear outside of core-drum. One should thus consider these references to be to stack position "infinity", meaning that they would be disk references no matter how far core-drum were extended, or simply "highly anomalous", and not considering them at all. The latter course, which we have chosen here, is equivalent to ignoring the effect of these pages on the extension stack ordering, and considering them to be outside the domain of the extension stack, that is, in core-drum. The effect of removing references to such pages on \( MHBF(n) \) is easily calculated. Starting from equation 3.4, we multiply both sides by \( MHBF(n) \), and obtain

\[
\sum_{t = E+1}^{\infty} r(t) \rightarrow MHBF(E) = MHBF(n) \sum_{t = n+1}^{\infty} \frac{r(t)}{E+1}
\]

If \( E \) is greater than the deepest position in the extension stack to which any of the anomalous pages ever migrates, the only place in this equation where anomalous pages are counted is \( MHBF(n) \). The effect of removing the anomalous fetches from this quantity is simply to scale it proportionately to the number of page fetches to be not considered. That is, if \( T \) page fetches (other than the "startup transient" fetches of section 2.2) were observed, \( A \) of them to the anomalous pages,

\[
\frac{MHBF(n)}{MHBF(n)}_{\text{adjusted}} = \frac{MHBF(E)}{MHBF(E)}_{\text{adjusted}} = \frac{T - A}{T}
\]

This ratio was observed to be between .92 and .96 for the measurements "dtm 23" and "dtm 21" displayed here.
Summarizing, the anomalous drum-abhorring pages create an inaccuracy of about 4 to 8 percent in the headways and headway ratios calculated from the measured data.
3.4.3 Inaccuracies Resulting from List Deletions

These inaccuracies result from a design decision to implement a simple, fast list-maintenance algorithm, as correct treatment of these deletions would require a fairly time-consuming technique. Hence, we proceed to analyze the extent of the inaccuracies resulting from this inaccurate treatment of deletions.

Recall from Chapter 2.2 that the deletion of pages in core-drum does not affect the ordering of the extension stack. Such a deletion implies that a fetch into core-drum will occur with no corresponding ousting. As this is in fact what happens, there is no inaccuracy involved with core-drum (or "out of list") deletions.

The deletion of a page from the extension stack creates a "moving anomaly", as discussed in section 2.2. All references to pages in positions in front of the anomaly (which occupies the position of the deleted page in the extension stack) are tallied correctly. The first reference to a page behind the anomaly is tallied incorrectly, because we chose not to record the anomaly. It is recorded as being one page closer to the top of the extension stack than it should have been. However, the anomaly now moves down to the position of that fetch. The situation is now the same as had the page just referenced been deleted. References in front of that position are tallied correctly, and exactly one reference behind it is tallied incorrectly, and the anomaly moves down.

We proceed to analyze the motion of such an anomaly down the extension stack. In the worst case, the page deleted was at the very top of the extension stack, and thus, the next reference is guaranteed to be tallied incorrectly. Probabilistically, this reference will be to that extension
stack position which is the mean of the distribution \( f(x) \), the measured
reference frequency distribution. There is a probability of \( q \) that this
reference will be as far down the LRU stack that a fraction \( q \) of the
weight of the distribution \( f(x) \) is below it. Now for the measurement "dtm
23", there were about 2000 in-list deletions per 50,000 in-list reads.
This means that there was one deletion per 25 reads. In order to calcu-
late the probability that a deletion anomaly is past a certain depth in
the extension stack by 25 reads, we consider the experiment, tried 25 times,
of encountering a read at least that far down the stack. The probability
of at least one success in 25 tries is one minus the probability of
exactly 25 failures, or \( (1-(1-q)^{25}) \). For \( q = .1 \), i.e., the ninety per-
centile point of \( f(x) \), it is .93. For \( q = .05 \), it is .72. Hence, by the
time the next deletion is recorded, it is quite likely that the anomaly
generated by the previous deletion is quite far down the extension stack.
Hence, for the upper portion of the extension stack, the effect of dele-
tions do not cumulate. Hence, each deletion generated an inaccuracy of
one stack position for each read behind it, but the corresponding anomaly
moves sufficiently rapidly down the extension stack that the effect of
later deletions are independent. Thus, for the upper portion of the
stack, the result of these deletions is a total uncertainty of one stack
position, a negligible amount.

The above reasoning correctly implies that the anomalies resulting
from deletions accumulate at the lower reaches of the extension stack,
and that fetches from these regions have a large cumulative error. In order to analyze this effect, we construct a queueing theoretical model of list deletions. A deletion anomaly causes the first fetch to a position behind it to be off by one position. Several deletion anomalies in the extension stack cause the first fetch behind them to be off by that many positions (the number of deletion anomalies). However, once this first fetch has occurred, the next fetch from this position will be off by one less position, and so on, until all of the deletion anomalies have moved behind the position in question, and fetches from this position are tallied correctly. Thus, we may construct the following interpretation: The deletion anomalies in front of position p form a queue. Each fetch behind position p "services" one request, i.e., removes one item from the queue. The rate of arrivals to this queue, in the worst case (all deletions from the very top of the extension stack) is the rate of deletions. The rate of service is the rate of references to stack positions behind position p. The length of the queue is the number of outstanding anomalies in front of position p, which is the total error in stack position by which fetches from position p will be tallied. Assuming exponentially distributed arrivals and services, with respective means $\lambda$ and $\mu$, the average queue length at position p is known to be $L = 1/(1-\lambda/\mu)$ from queueing theory. $\lambda/\mu$, the ratio of arrival rate to service rate, is the total number of deletions (assuming the worst case) divided by the number of references past position p, both immediately obtainable from the measured data. As there were 2000 total deletions*, queue length approaches infinity at the point in the extension stack where 2000 references were counted below that point. This point is at 3650 pages depth. At 500 positions

*In measurement "dtm 23".
before this, queue length is down to 4. At only 100 positions before it, queue length is 20. Hence, up to 3000 pages, the effect is negligible. At positions below that point where there are 2000 page fetches recorded below, the queue grows faster than it is serviced. At the time the experiment is stopped, the number of deletion anomalies remaining in queue in front of position \( p' \), where \( p' \) is beyond the 2000 reference point in the extension stack is the total number of deletions (in the worst case) minus the number of references beyond position \( p' \). As both of these quantities presumably grow at a constant rate, the average error in stack position of a recorded reference to position \( p' \) is one-half this queue length. This allows us to reconstruct an approximation of the correct \( r(x) \) and \( f(x) \), and then all of the resulting curves, by recreating a better, more accurate \( r(x) \), \( r'(x) \) as

\[
r'(x) = r(x) \text{ for } x < 2000 \text{ pages}
\]

\[
r'(x + \frac{2000-t(x)}{2}) = r(x) \text{ for } x \geq 2000 \text{ pages}
\]

where \( t(x) = \sum_{y=x}^{x} r(y) \)

This implies that at the very tail of the distribution, there is a stack position inaccuracy of \( 2000/2 = 1000 \) positions. At a stack depth of 5000 positions, there is an inaccuracy of 500 positions. This does not seriously affect the shape of the exception ratio and MHBFF curves in the region of interest as one can see from figure 3.6. We have re-plotted here figure 3.1 and corrected as above.
3.4.4 Other Inaccuracies

Another possible source of inaccuracy was the forced oustings of pages to disk directly from core, not due to global transparency to the paging device. A close inspection of "try_to_write_page" in Appendix A reveals that pages which should be ousted from core to the paging device (the drum) are occasionally ousted to disk, because there is no room on the paging device. This action avoids recursion in the process of finding a free core frame, as the latter process would otherwise possibly involve ousted pages from drum, which could require finding a free core frame. Although we do not have data on the frequency of this occurrence on the days of the experiment, we have observed Multics at other times, and the percentage of disk writes caused by such oustings is less than a tenth of a percent of all disk writes. It is true of Multics that the ratio of reads to writes remains fairly constant. Each read corresponds to one page fetch, and each fetch must be accompanied by an ousting at some time. Hence, 'forced' oustings must be a similarly small percentage of all oustings, and not a significant effect.
3.5 Correlation between dtm 21 and dtm 23

Observing figure 3.2, the correlation between the two plots is fairly remarkable. Within any reasonable accuracy for what is meant to be used in engineering approximations, these curves represent a measurement of the same quantity.
3.6 Our Result and the Linear Model Measurements

Our mean headway curve (figures 3.1, 3.3) shows distinct differences from the curves given in S1 for the mean headway function of the Multics system. We can attempt to rationalize these differences by understanding the nature of the user load during the two different experiments.

The measured mean headway between disk page faults, in terms of virtual memory references per disk fault, was between two and three times the figure measured in S1. What is more, the slope of the two curves differs, ours starting out at almost six times the slope of the curve S1. We attribute this to differing values of \( \lambda \) in equation 3.13, in terms of the model proposed in this experiment. More specifically, the 'tightness' of working sets was greater for our experiment, and the number of distinct users was fewer, causing even greater tightness of the system's "combined working set" at any time. The measurements given in S1 were made during a day of very heavy system usage, in August 1972. User load at this time consisted primarily of systems programmers engaged in program development, an activity which references vast extents of libraries, tools, and specialized procedure and data. These users were also operating without economic restriction, and thus had little incentive to minimize the resources used by their activities. Our experiments were conducted at a time when some of the Multics user load had shifted to the Honeywell 6180 Multics system, in a state of development at that time. All of the systems programmers had moved to the new machine at the time of our experiments, and the remaining user load was quite light, consisting of the M.I.T. academic community. The lightness of user load also implies a smaller number of distinct users.
The mean headway curves which can be extrapolated from our measurements may be viewed as a function of user load/system working set tightness, giving rise to the family of graphs of figure 3.7. From this, it can be seen that a linear region on a curve corresponding to a large system working set can correspond to a non-linear region on a graph corresponding to a smaller system working set, and the latter will rise faster than the former. If one draws the line C C' corresponding to the core-drum to disk boundary, both the differences on measured headway and slope differences can be more readily understood.

Another factor which gives rise to the family of graphs in figure 3.7 is the transient response of the experiment. As the length of the LRU extension stack grows, so does the observed value of \( \lambda \). Especially when user load is light (smaller number of disk references per hour), it takes longer to develop curves of low \( \lambda \) than high \( \lambda \), and this was the case in our measurements. Hence, it is possible that a more extensive measurement could have allowed a curve of higher apparent \( \lambda \) to result.
Figure 3.7 Family of Headway Curves of Differing $\lambda$
Chapter 4

Conclusions and Suggestions for Future Research

4.1 Conclusion

The most general and useful conclusion that can be drawn from this thesis is that increased primary memory size decreases page fault overhead in virtual memory systems very sharply as it grows, the decrease being much greater than proportional to the increase in memory size.

We hypothesize that the reference patterns observed, and the headway functions derived are characteristic of a large-scale computer utility being used by an academic community through interactive consoles. The data being referenced on Multics was accessed through a virtual memory mechanism; were it accessed via explicit disk requests on some other type of computer utility, we expect to see the same patterns and headway functions.

The most specific and concrete result which we have arrived at is a measurement of the mean headway function for Multics, showing how page fault overhead decreases as primary memory size approaches $4 \times 10^8$ bits.
4.2 The Paging Model Suggested

The mean headway function $\text{MHBPF}(x)$, where $x$ is primary memory size in pages, may be expressed as a polynomial in $x$,

$$\text{MHBPF}(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots$$

(1)

Saltzer's measurements suggest that

$$\text{MHBPF}(x) = \alpha x$$

(2)

is an adequate characterization of the paging behavior of Multics in the range $0 \leq x \leq 1.3 \times 10^8$ bits. Our experiment shows, for the particular measurements we made, that the quadratic term in (1) becomes significant at $x = \text{approximately} 1.0 \times 10^8$ bits. The trends indicated by figures 3.1 and 3.4 suggest that higher terms become significant as $x$ is increased further. This would be consistent with the observation made about arbitrary increase in primary memory size on a virtual memory system made above. Belady and Kuehner (23) assert $\text{MHBPF}(x)$ to approximate $\alpha x^k$ for 'real life programs'. In measurements made on IBM 364/44X and System 360/67 machines, they found $k$ to take values 'in the vicinity of 2'.

This model also can be described by the general representation of equation (1) above. These observations also suggest, as does figure 3.2, that

$$\text{MHBPF}(x) = (\alpha_0 - \beta) + \beta e^{\frac{\alpha}{\beta} x}$$

(3)

is in some cases a valuable approximation. The constant term becomes insignificant for sufficiently large $x$, and we may write

$$\text{MHBPF}(x) = \beta e^{\frac{\alpha}{\beta} x}$$

(4)
a very simple model which is very appealing. As was shown in Chapter 3, this model corresponds to a reference probability distribution

\[ p(x) = e^{-x/\lambda} \]  \hspace{1cm} (5)

where \( x \) is LRU stack depth and \( p(x) \) is the probability of reference to that position. This simple model of program behavior is particularly appealing, as it characterizes "program size" as a distribution. Denning (D1) has given the concept of 'working set' as a measure of program size, within a given time interval. Equation (5) is or a more specific class of program characterizations, expressing the 'size' of the program as a distribution. The parameter \( \lambda \) may be viewed as a 'radius of locality' of the programs running, expressing in some sense their 'tightness' or 'togetherness'. In this sense, \( \lambda \) is akin to the concept of working set.
4.3 Unanswered Questions and Future Directions

The most obvious extension of the work presented here is to extend upward the range of primary memory sizes for which the nature of \( MHBPF(x) \) is known. Although the techniques used in this thesis are completely extensible in this regard, it is not clear whether or not there is any value in such research beyond some point. If a certain amount of primary memory reduces secondary memory accesses to once an hour, for instance, the issue of secondary memory access time versus cost quickly takes precedence over primary memory cost versus secondary memory reference overhead. For instance, our measurements predict that another seven million words of core-drum would reduce disk references to once every two minutes. At this rate, the economic viability of a fast disk is a greater issue than the performance improvement resulting from more core-drum. For instance, a large \((10^{12} \text{ bit})\) slow (1 sec access time) store might be quite acceptable as a backing store.

Another area of research is to fully understand the program behavior patterns which are responsible for models of program behavior such as Saltzer's linear model and the model proposed above. We understand the working set model because we know that program loops, subroutines, etc., cause repeated reference to certain data items, and this behavior is somewhat extensible to larger views of programs. We do not know what "causes" the linear model, or other such models in this sense. We can understand "distribution" type models by the same considerations of 'spatial locality' and 'temporal locality' (M2) on which the working set model and the LRU replacement algorithm are based, but we have no insight into the basis for any particular distribution in program behavior.
A very major unknown limitation on the work of this thesis is the applicability of its result. No attempt has been made to determine precisely what aspects of Multics user behavior were responsible for the effects noticed. It is not even clear how specific the results of this thesis are to the LRU algorithm.

An interesting direction of research would be to perform a similar experiment on some LRU-managed system which does not utilize virtual memory. A large data management system utilizing an LRU-managed buffer pool might be such a system. The common aspects of user behavior accessing a large on-line data base might show through here, as it is the referencing patterns, not referencing methods which are interesting.

Systems such as IBM's OS/VS2 and VM/370, involving paged virtual memories and multiple address spaces provide a fruitful ground for comparison. In these systems, features of sharing and data addressing are quite different than Multics, but the amount of data to be addressed and the primary memory usage strategy are not altogether dissimilar.

An important direction to be pursued is that of repeating this experiment on Multics, reliably, many times, and determine day-to-day and hour-to-hour variations in the behavior of MHBPF(n). The thrust of this thesis was to develop and apply the techniques stated herein, and others must use these tools to correlate MHBPF (n) to whatever factors appear as influential.

An interesting issue is to relate the parameters \( \lambda \) and \( \alpha \) in all of these models of program behavior to other observable parameters of program behavior and system configuration. This would constitute a long step toward theoretical understanding of the behavior which underlies these
models.

Sekino (S2) shows the significance of \( \text{MBP}^T(x) \) in system performance calculations, particularly throughput and response time. Although our results may be used in these calculations, we have not pursued this course here.
Appendix A

A Structured Program Description of Multics Page Control

This appendix describes the functioning of the fault and interrupt driven mechanisms within the Multics virtual memory management algorithm as it existed in May, 1973, at the time of the experiment. Only the paths within the so-called 'Page Control' subsystem relevant to this thesis have been shown. This excludes some fairly complex mechanisms relating to error handling and the allocation of page tables. Within the paths shown here, however, this results in only a few small omissions.

The aim of this appendix is to familiarize the reader with the internal operation of page control to whatever depth is necessary for comprehension of the rest of this thesis, particularly Chapter 2 and Appendix B. To this end, we have provided a description on several levels.

The most detailed description of page control given here is an approximately "structured" program, in which we have functionally modularized page control into 14 small routines. We have taken the liberty of creating a new language in which to write this program, which we explain within. We feel that this language conveys the general class of manipulation described herein with a maximum of clarity and succinctness.

We have liberally renamed objects, substituting names which we feel are more mnemonic than the actual names used in Multics. We have also made minor modifications to control flow, and subroutinized routines which were not originally subroutines where we felt that clarity would be aided. In any case, the algorithm as given is essentially identical to the actual assembler-code algorithm at the time of the experiment, with
respect to state, sequencing, and side effects.

The plus sign (+) in the left-hand margin denotes references to routines explained in detail within.
A Brief Overview

Multics manages both core and drum (the latter known as the "paging device", or "pd") by approximations to the least-recently-used algorithm. Two lists, the core used list and the paging device used list are maintained for this purpose, the top of each list designating the least recently used page (which is the best choice for replacement), and the bottom of each list designating the most recently used page (which is the worst choice for replacement) on the respective devices. How these lists are maintained can best be learned by reading the program that we have provided. The core used list contains logical descriptions of core frames, including pointers to descriptions of logical pages and/or paging device records when such entities may be associated with the core frame. Similarly, the paging device used list contains logical descriptions of paging device records, including pointers to descriptions of logical pages and core frames, when such entities may be associated with the paging device record.

Multics tries to maintain copies of the most recently used P pages (where P is the size of the paging device, in records) of the storage system on the paging device. The most recently used C pages (where C is the size of core memory in page frames) are to be in core, as well. (It is assumed that C is less than P.)

Thus, pages being ousted from core may be written to the paging device, even if a good copy exists on disk. This fact should be kept strongly in mind when reading "try_to_write_page". Except for the case where the paging device has no copy, pages which were identical to pages in secondary storage are never written out. Pages of zeros are never
written out, but their logical description is so modified that they are created in core when faulted on.

The processor hardware maintains usage information about a logical page in a hardware descriptor. Specifically, the occurrence of usage and/or modification is noted in the descriptor.

A page fault is resolved by finding a page of core into which to bring the page, and bringing it in. Finding a page of core consists of reorganizing the core used list to reflect the latest usage information, and finding the least recently used page frame, and using it. Pages which have been marked as modified cannot be claimed in this way, but are written out. When the writing is complete, at some future time, the page will be in the same state as a page which has not been recently used or modified, and will be claimed in the handling of some future page fault. Note that this 'writing' consists of initiating the physical operation, but not waiting for it to complete. It is at this writing time that secondary storage is allocated, and pages containing zeros are noted. It is at the time that zero pages are noted and that secondary storage is deallocated.

At the beginning of page fault handling, housekeeping is performed on the paging device, which consists of trying to insure that at least ten records are either free or in the process of being freed. This is done by removing as many of the least recently used pages on the paging device as necessary. When a page is so moved, it is checked (via software-maintained switches) to see if it is identical to a copy on disk. If so, it may simply be deallocated from the paging device. If not, a sequence known as a read-write sequence (rws) must be performed. This sequence consists of allocating a page of core to be used as a buffer,
reading the page into it from the paging device, writing it to disk, and deallocating the paging device copy. The core buffer is then freed.

A page fault which occurs on a page for which a read-write sequence is in progress causes an event known as an rws abort to occur. The freeing of the buffer page and the paging device page are inhibited, and the buffer page is used as the core copy of the page, and the fault is resolved.
An Explanation of the Language
Used to Express this Description

The language which we have used to describe Page Control is a bastardization of PL/I, with new primitives for some basic operations (enqueue, masked procedures, etc.) and an Algol 68-like formalism for representing relationships among structured entities.

Underlined words are language keywords. Lower-case identifiers represent names of subroutines, functions, or labels. Identifiers beginning with an upper-case character represent references to cells, which will be described below. Statement syntax is essentially the same as PL/I, but "=" is used for assignment, and "=" is used to test equality. There is no lexical nesting of procedure or begin blocks.

A program consists of begin blocks, entered from the outside world in some unspecified way, procedures and functions, and declarations. declare (dcl) declarations may appear anywhere, including outside of blocks, and are global in scope. They define the class and type of variables, and the types of Objects used by the program. local declarations appear within blocks, and define a local scope of variables, identical to that produced when a variable is used as a formal parameter in a procedure or function.

The point of this language is to associate cells with values. The domain of values is the space of Objects. Objects are unique. Two cells have equal values if and only if their values are the same Object.

There are three classes of Objects: primitive Objects, structured Objects, and set Objects. Within each class, there are different types
of Objects. Objects have no names. Only primitive Objects can be referred to explicitly, i.e., other than by reference to a cell having the desired Object as a value, or a function returning the desired Object.

Primitive Objects can be of three types. The first is boolean. There are exactly two boolean Objects. One can be referred to explicitly as true, the other false. The second is arithmetic. There is a first-order infinity of these objects, which are actually the integers. They can be referred to explicitly as 75, 16777216, -283, etc. The third is literal. They are simply arbitrary primitive Objects, whose only useful property is their uniqueness. They can be referred to explicitly as "foo", "bar", "no stuff", etc. They are not character strings in any sense, but simply unique primitive Objects of type literal.

Structured Objects consist of a finite number of cells. Any cell can have as a value only one type of Object (implied is one class, as well). These cells are called components of the Object. These cells do have names, and they are specified in a declaration which describes the concerned type of structured Object.

Set Objects consist of an ordered set of Objects of the same type and class. All references except enqueue and dequeue, however, consider the set Object as unordered. One can add to or enqueue to a set Object, remove from or dequeue from it, ask if a given Object is a member of it, or cause a cell to be assigned successive values, each value being a different Object in the set Object, in no particular order.

Variables are the other type of cell. A variable can hold only one class and type of Object, just like the other type of cell, the structured Object component.
**Assignment** (performed by ":=" operation in do statements and assignment statements) consists of replacing the value of a cell with another value, i.e., changing the value of the cell. The object which was the previous value is neither changed nor destroyed in any way.

**Binding** consists of saving the value of a variable when a procedure, function, or begin block is entered, and restoring it when it is exited. The latter operation is called unbinding. All assignments and bindings made between the time a variable is bound and the corresponding unbinding have a transparent effect when the block performing the binding is exited. A local declaration of a variable in a block causes such a binding to take place for that variable when the block is entered, and the corresponding unbinding. Binding also takes place for variables used as formal parameters to procedures and functions. In this case, after the old value is saved, the value of the corresponding formal argument is assigned to the variable. Hence, all calls may be seen as "call by value".

To refer to an Object, one can either refer to a cell containing it, or, if it is primitive, one can refer to it explicitly. To refer to a variable, simple state its name. To refer to a component of a structured Object, state its component name, an open parenthesis, a reference to the structured Object, and a close parenthesis.

An assignment is a reference to a cell, ":="; and a reference to an Object of the same type and class declared for that cell.

Variables need not be declared. The default class of any cell is structured, with a type the same as its name. The syntax for a structured Object type declaration is as follows:
\{declare\} \textbf{structured} Foo (compdcl-1, compdcl-2,\ldots compdcl-n);

\{ \} = \text{optional} \quad \{ \} = \text{select one}

The \texttt{compdcls}, or component declarations, are of the same syntax as variable
declarations, except that the name is the name of the component, and the
optional keyword \texttt{variable} is illegal.

The syntax for a variable or structured Object component declaration
is as follows:

\{\texttt{declare}\} \quad \{\texttt{variable}\} \quad Foo \quad \{\texttt{type}\} \quad \{\texttt{objtyp}\}

where \texttt{objtyp} is either \texttt{boolean}, \texttt{literal}, \texttt{arithmetic}, any structured Object
type named in a structured Object type declaration, or \texttt{set} \texttt{objtyp}, where
\texttt{objtyp} is, recursively enough, any possibility named in this sentence.

\texttt{local} declarations only name their variable, although they can declare
its type as well.

\texttt{do} statements differ from PL/I in that any cell can be used to the
left of the ":=", not necessarily variables. The particular form "\texttt{do}
Foo::= \texttt{range Bar}" means that the value of Bar is a \texttt{set} object, and the
\texttt{do} is to iterate over each Object therein, in no special order.

The special constructor function \texttt{construct} is used to create new
structured objects. The syntax of a reference to it is

\texttt{construct} Foo (compname-1:object-1,compname-2:object-2\ldots ),
whose value is the new Object.

The unique Object "null" can be used as a value of any cell. It has
all types and classes.

The predicate \texttt{void} takes as an argument a reference to a set Object,
and returns \texttt{true} or \texttt{false} (\texttt{boolean} Objects), depending on whether or not
it is empty. The operators "=" and "\neq" may be used to test if two references are equal, i.e., refer to the same Object. An appropriate boolean Object is returned as a value. The operators "or", "and", and "not" operate on boolean Objects in the obvious way. The conventional arithmetic operators operate upon arithmetic Objects, returning an arithmetic Object with the expected value.

if statements have as their predicate a reference to boolean Object.

A call statement consists of the word call followed by either a procedure name and an optional argument list or a complex function reference and an argument list. An argument list is a parenthesized list of (possibly zero) references to Objects separated by commas. A complex function reference is a function reference to some outside-of-the-language function which will return as a value a procedure, which one depends on the arguments to the function, which will be called by the call statement, with the arguments to the call.

The evaluation of arguments in or and and is conditional, as in Lisp 1.5 (M3) and proceeds from left to right.
A Program to Find the Man Who Owns the Black House, and Have Him and His Father Switch Houses

```plaintext
declare structured Person

  (Father type Person,
   House);

declare structured House

  (Color literal,
   Owner type Person);

declare Son Person, House2 House;

declare Brooklyn set House; /*assumed to be initialized*/

switch_houses: begin;

  do House := range Brooklyn; //search the set "Brook-
    //lyn"
    if Color(House) = "black" then do; //found him
      House2 := House(Father(Owner(House))); //find the other house
      Son := Owner(House); //remember who is the
        //son
      House(Son) := House2; //Son now owns House2
      Owner(House2) := Son;
      House(Father(Son)) := House; //Father owns house
      Owner(House := Father(Son);

  return;

end;

end;
```

A Top-Level Programmatic View of Page Control Activity

A page fault causes the following:  
(page_fault)
The paging device is housekept.
Transient conditions such as i/o in progress or an rws on
the faulted page are noticed and handled.
A free page is claimed, and the faulted page is read or created
into it.
If i/o was started, the page is waited for.

Finding a free page consists of the following:  
(find_core)
The core used list is searched for a good candidate.
Recently used pages are not good candidates. They are skipped, and
re-judged as not-so-recently used for next time.
Pages which have been modified (stored into) cannot be claimed now.
They are written out, and re-judged as not to have been modified.
A page which has not been modified, and has been used approximately
less recently than any other page, is pre-empted from its core
frame, and this core frame is the new free page frame.

Writing a page out consists of the following:  
(write_page)
The page's contents are checked, and if all zeros, the page is
flagged as not needing to be read or written - No writing takes
place, and disk and paging device space allocated to the page
are freed.
The page is given a residence on disk, if it does not already have
one.
The page is given a residence on the paging device, if it does not
already have one, and one is available.
The page is written out to its residence on the paging device, if it
has one, otherwise to disk. The completion of i/o is not waited
for.
Housekeeping the paging device consists of the following: (get_free_pd_record)

An attempt is made to insure that there are ten paging device records free or being freed, which is done as follows:
The pd used list is searched for a good candidate to pre-empt.
The search is made starting at the least-recently used pd record.
Records which contain pages in core are recently used. They are rejudged as such and skipped.
Records containing pages identical to pages on disk are acceptable.
The pages in them are pre-empted, and the record is now free.
Other records have to be written back to disk, which is done by performing a read-write sequence (rws) on them.

Performing a read-write sequence on a page consists of the following: (start_rws,rws_done)

A free page of core is obtained.
The page is read into it from the paging device.
When the read is completed, the page is written out to the disk.
When the write is completed, the page of core and the paging device record are freed.
A page fault on the page involved in the sequence at any point during it causes the sequence to be aborted at the next complete operation in the sequence, and the core page is used as the page's home in core.
A Top-Level Description of the Objects Used by Page Control

A Page Object is the logical description of some page of the storage system, as opposed to a page frame on some device.

A Descriptor Object, in actuality a "page table word", is the physical descriptor by which a processor accesses a page. It contains a core address, usage bits, and a bit which causes a fault when off.

A Coreadd Object describes a physical core block. It describes the status of this block, including, implicitly, its position in the core used list.

A FDrec Object describes a paging device record, or frame. It describes the status of this frame, including, implicitly, its position in the paging device used list.

A Devadd Object represents a physical disk or drum address, and its contents. Included in this object is an identification of the device on which this page frame resides.

An Io-status Object is a hardware-generated object, which describes an input-output operation which has completed.

An Io-program Object is a sequence of commands for the system i/o controller to give to an i/o device. It specifies the type of operation required, the record within the device concerned, and a core address concerned.
A Trace-Datum Object is a recorded datum of information about traffic between disk and core-drum, for the purpose of the thesis experiment.
/* Description of the structured Object types used by Page Control. */

Recall that the default type of a structured Object component is the same as its name. */

dcl structured Page
(Descriptor, Devadd,
Coreadd,
PDrec,
Event literal,
lo_in_progress boolean,
On_pd boolean,
Uired boolean,
Gtpd boolean);

// Represents a page of some segment of Multics, as opposed to
// a page of core or some device.
// The hardware descriptor by which processors access the contents of Page.
// The physical disk or pd address from which Page should be
// read or written to. If On_pd is true, is a pd address. Otherwise,
// it is a disk address. A Devadd of "null", however, represents a
// page full of zeros.
// The core frame associated with this page. Valid only when
// Addressable (Descriptor (Page)) is true.
// If On_pd is true, this is the pd record used by Page.
// Some literal quantity unique for each page. Used to identify the
// occurrence of events associated with this page in interprocess signaling.
// Truth indicates i/o in progress, or at least not known to have
// completed, on Page.
// Specifies that Page has an allocated PD record, namely PDrec (Page).
// Indicates that Page must always remain addressable.
// Indicates that Page is forbidden to xo on the pd, for reliability reasons.

dcl structured Descriptor
(Phys_Coreadd arithmetic,
Addressable boolean,
Usage boolean,
Modified boolean);

// Represents a page table word (ptw), the physical descriptor by
// which processors access a page.
// The physical core address occupied by the page to which this Descriptor
// belongs. Valid if and only if Addressable is true.
// Truth allows Phys_Coreadd to be used by the processor. Falsity
// causes the procedure "page_fault" to be executed.
// Set by the hardware whenever this Descriptor is used,
// or more accurately, fetched into the associative memory.
// Set by the hardware whenever a store-type operation is
// performed using this descriptor, or an associative memory
// copy thereof.

dcl structured Coreadd
(Page,
Phys_Coreadd arithmetic,
Next_type Coreadd,
Previous_type Coreadd,
lo_read_or_write literal,
Rws_in_frame boolean);

// Represents a core page frame.
// "null" represents an unallocated page frame. Otherwise, the
// page contained in this frame. This is only for normal page-holding
// use, not rws's.
// The physical core address represented by this frame.
// The next more recently used core frame.
// The next least recently used core frame.
// if lo_in_progress(Page(Coreadd)) is true, or Rws_in_frame,
// tells which direction of i/o is being performed.
// Signifies an rws in progress in this frame.
del structured Devadd
(Device literal, 
Phys_Devadd arithmetic); 
// Represents a physical device address. 
// Identifies a secondary storage device. 
// Identifies a physical record number on some device.

del structured PDrec
(Page, 
Diskaddr type Devadd, 
Devadd, 
Coreaddr, 
Next_pd type PDrec, 
Previous_pd type PDrec, 
Event literal, 
in_use boolean, 
Rws_in_progress boolean, 
Incore boolean, 
Modified_from_disk boolean, 
Abort_flag boolean, 
Abort_complete boolean); 
// Represents a paging device (nd) record. 
// If in_use is true, describes the Page on this record. 
// If in_use is true, describes the disk address occupied by our page. 
// The physical device address of this record. 
// When Rws_in_progress is true, describes the core frame 
// being used as an rws buffer. 
// Describes the next more recently used pd record. 
// Describes the next least recently used pd record. 
// A unique literal associated with this pd record. Used to identify the 
// latter in interprocess signaling. 
// Tells if this pd record is in use or free. 
// Signifies that an rws or rws abort is in progress in this pd record. 
// Signifies that the page in this pd record is in core right now. 
// Used for maintaining the LRU ordering of the pd used list. 
// Truth indicates that the pd copy of page is different 
// than the disk copy. 
// Turned on to start an rws abort by some process faulting on an rws'ing 
// Page. 
// Signifies that post_save (a.o.v) has aborted an rws, and a cleanup 
// by rws_abort (a.o.v) is expected.

del structured lo_program
(Direction literal, 
Phys_Devadd arithmetic, 
Phys_Coreaddr arithmetic, 
Next type lo_program); 
// A portion of a channel program. 
// Indicates read or write. 
// Physical device address involved. 
// Physical core address involved. 
// Next program in channel queue.

del structured lo_status
(Phys_Devadd arithmetic, 
Phys_Coreaddr arithmetic, 
lo_program, 
Coreaddr); 
// Represents a completed I/O operation to an 
// I/O control routine. 
// Identifies the physical device address involved. 
// Identifies the physical core address involved. 
// The lo_program object which initiated the operation 
// causing this status to be generated. 
// The Coreaddr object associated with Phys_Coreaddr. Although 
// not actually present here, the one-to-one mapping between 
// Coreaddr's and valid Phys_Coreaddr's lets us use this here for 
// clarity.

del structured Trace_datum
(Devadd, 
Type literal); 
// An item of trace data for the experiment. 
// The disk address concerned. 
// The direction and description of logical motion. 
// "read" - a page fault to disk or an rws abort 
// "write" - an rws initiation 
// "virtual" - an outing from core to disk
// "pd_virtual" - an ousting from the pd to disk
// "delete" - the deletion of a page.
The Global Variables Used by Page Control

dcl
Page_table_lock literal, A quantity used to insure that only one pro-
cessor at a time is in page control. A pro-
cess desiring to "hold" this lock loops con-
tinuously until it is unlocked, and then
locks it.

CoreTop type Coreadd, The least recently used Coreadd Object.

Writes_outstanding arithmetic, The number of write operations started which
have not yet been known to complete. Used
as a heuristic to call post_any_io.

Rws_active_count arithmetic, The number of read-write sequences which have
been initiated and not yet known to be com-
pleted.

Number_of_free_pd_records arithmetic, The number of paging device records free or
in the process (rws) of being freed.

Top_of_pd_used_list type Pdrec, The least recently used Pdrec Object.

Channel_Seq type Io_program, The executable queue of i/o programs for a
disk or drum.

Experiment_active boolean, Tells if metering experiment is in progress.

Trace_queue set Trace_datum, The total of all trace data accumulated by the
experiment.
Undocumented Routines Referenced in this Program

page_wait (literal)
Suspended the calling process until a call to notify is made with the identical literal.
page_wait also unlocks the page_table lock once the traffic control databases are locked.

notify (Literal)
Causes any process which called page_wait with the identical literal to be resumed.

clear_associative_memory
Causes all processors to clear their associative memories. This routine does not return until all processors have indicated that they have done so. Used to force access turnoffs and modified bit turnoffs to take effect.

allocate_disk_record() Returns an unallocated Devadd Object. Marks it as allocated.

relinquish_disk_space(Phys_Devadd)
Marks a Devadd Object as unallocated to allocate_disk_record.

start_io(Io_program)
Starts a channel executing an i/o program.

thread_to_top(Coreadd)
Changes core used list and value of CoreTop such that Coreadd is moved to the top of the Core used list (least recently used).
CoreTop now = Coreadd.

thread_to_bottom(Coreadd)
Changes core used list and value of CoreTop such that Coreadd is moved to the bottom of the core used list (most recently used).
Next (Coreadd) now = CoreTop.
decl Scu_data Virtual_reference;
dcl structure Virtual_reference (Page, Segment);

page_fault: begin
  mask;

  local Page;

  set_lock Page_table_lock;
  Page := Page (Scu_data);
  do while Number_of_free_pd_records < 10;
    call get_free_pd_record;
  end;

  if Addressable (Descriptor(Page))
    then unlock Page_table_lock;
  else if lo_in_progress (Page)
    then call page_wait (Event(Page));
  else if On_pd (Page) and Rws_in_progress(NReq(Page))
    then do;
  end;

  call rws_abort (Page);

  // This procedure is transferred to when the
  // hardware determines that a reference has been
  // made through a descriptor whose Addressable bit
  // is false. Return from this procedure causes
  // a second attempt to make that reference.
  // This procedure is entered in such a way
  // that all external (i.e., I/O, etc) interruptions
  // are disabled when it is entered. They are reenabled
  // when it is exited. This is because such
  // interruptions might try to lock the page-table lock.

  // Prohibit access to page control by other
  // faulting processors.

  // Determine from processor state at fault time which
  // page was faulted on.

  // Housekeep the paging device - try to have some free
  // pd records for the find core calls which will
  // surely follow.

  // It is possible that we took a page fault while the
  // page table was locked, and the process holding the
  // lock brought the page in. Exit if objects true.
  // It is possible that we took a page fault on a page
  // which some other process has started bringing in.
  // Wait for it.

  // The system routine page_wait causes the
  // suspension of the calling process until
  // some other process calls the routine notify
  // with the identical Event with which page_wait
  // was called. Page_wait also unlocks the
  // page table lock once he has locked his data base.

  // If this page is on the paging
device, it is possible that a read-write
// sequence may be in progress for it.
// It must be aborted.
// Abort the rws, or possibly clean up an
// already complete abort. Unless we are
// cleaning up, we will wait for it.
If Rws_in_progress(PDrec(Page)) // If we have cleaned up, we are 
   // finished. If we started an abort or 
   // noticed one in progress, we must wait.
   then call page_wait(Event(PDrec(Page))); // Wait for the abort to 
   // Complete.
end;
else do; // Normal case - we must bring Page in.
   call read_page(Page); // Start read-in of Page. If page is empty 
   // (all zeros), or was on a fast device, we may be done.
   if lo_in_progress(Page) // If real I/O was started, and not finished,
      then call page_wait(Event(Page)); // we must wait for post-page 
      // to post Page.
      else unlock Page_table_lock; // Page was all zeros or on fast device. 
      // All done.
end;
return; // Restore machine state at time of fault. Retry 
// faulting reference.
read_page: procedure(Page);

local Coreadd;

if On_pd(Page)
    then if Rws_in_progress(PDrec(Page))
        then error;

        Coreadd := find_core();
        Page(Coreadd) := Page;
        if Devadd(Page) = "null"
            then do:

                Page.Coreadd(Coreadd) := 00000000; // Make it all zeros.
                call make_accessible(Page);
                io_in_progress(Page) := true;
                io_read_or_write(Coreadd) := "read";
                end;

                call device_read(Devadd(Page), Phys_Coreadd(Coreadd)); // Start the read.
                if On_pd(Page)
                    then do:

                        incore(PDrec(Page)) := true; // has a page in core (recently used).
                        call pd_thread_to_bottom(PDrec(Page)); // This line is not in the actual code.

                        end read_page:
                    end;

                 // This procedure is responsible for
                 // causing a faulted page to appear in core.
                 // If I/O is necessary, it is initiated. If not,
                 // a page of zeros is created.

                 // Although this check is made in page_fault,
                 // it is conceptually important that it be made here,
                 // for read_page may be called by other system
                 // functions. If an I/O is in progress on Page,
                 // we cannot read it, and our caller must either
                 // give up or initiate abort proceedings.
                 // Allocate a page of core for the page.

                 // Indicate that this page belongs here.
                 // A null devadd indicates that a page is defined
                 // to contain zeros.

                 // Indicate that this frame belongs to Page.
                 // Reset fault bit in the descriptor, allowing
                 // processors to reference Page.

                 // Page has non-null devadd must be read in.
                 // Indicate that Page has I/O in progress. It is
                 // important that this be done before the I/O is
                 // initiated, so that the I/O routine need not
                 // start if a fast device finishes early.

                 // Let pd management know that this page

                 // has a page in core (recently used).
                 // This line is not in the actual code.

                 // Its absence constitutes a bug which was found
                 // later. It helps maintain the LRU ordering
                 // of the PD used list. */
find_core

find_core: function();

dcl CoreTop Coreadd;

local Frame type Coreadd;
local Page;

local lo_skip_counter arithmetic,
loop_counter arithmetic;
lo_skip_counter := 0;
do loop_counter := 0,
repeat loop_counter < 131072 ;

\ +

\ This function implements the Multics core
\ page replacement algorithm, it is called to
\ housekeep core, and return one free Coreadd Object.
\ Its basic data base is the core used list, which
\ is the ordering of Coreadd Objects defined by the
\ sequence of Next components of Coreadd Objects.
\ The variable CoreTop has as a value the least
\ recently used Coreadd. Next(CoreTop),
\ Next(Next(CoreTop)), and so on, are Coreadd
\ Objects having been increasingly recent use. The core
\ used list is circular, so Next (most recently used
\ Coreadd) = CoreTop. The algorithm is due to
\ Corbató.

\ Initialize check for excessive looping.

\ We search the used list as long as necessary.
\ The default is to crash Multics:
\ We will always return a free core, or die.
\ We use this loop to prevent endless
\ looping.
\ If this routine has caused many writes, see if
\ some have completed since we started looping.
\ (Remember that interrupts are masked).
\ Reinitialize i/o check. We have just done what
\ this check could ask us to do.
\ A page of code is required.

\ We check the supposedly least recently used page to
\ see if it is entirely unallocated (could happen by
\ use for an I/O, or a page deletion). If so, we take it.
\ This will be the page frame.
\ This is the most recently used, and the
\ next least recently used the least. Notice that
\ this common operation is trivial only because of the
\ circularity of the core used list.
\ Return this core frame as usable.

\ If, in the next line, we are going to skip this
// frame because of I/O, other than an I/O going on,
// meter the times that we have done so.

then lo_skip_counter := lo_skip_counter + 1;

if lo_in_progress (Page (CoreTop)) // See if we must skip this frame due to I/O
    or Rwls_in_frame (CoreTop) // in progress there.
    then if lo_skip_counter < 100 // if we have skipped a large number of pages due to
        then do;
            call post any io;
            io_skip_counter := 0; // Reset this high-water-mark.
        end;
        // repeat the loop, trying this last I/O-skipped
        // page again.
        else CoreTop := Next (CoreTop); // Skip over this core frame, consider Next to
            // be LRU.
        else if Wired (Page(CoreTop)) // Skip over pages never claimable.
            then CoreTop := Next (CoreTop);
        else if Usage (Descriptor (Page(CoreTop)))
            then do;
                Usage (Descriptor (Page(CoreTop))) := false;
                    // if so, reinitialize check for
                    // next time, and skip this
                    // page, making
                    // it the most recently used. This
                    // Usage bit is set true by the
                    // hardware when the descriptor is used.
                CoreTop := Next (CoreTop); // Skip over it.
            end;
        else do;
            // At this stage, the page at the top of the
            // core used list has not been recently used.
            // It is a prime target for replacement. We will
            // see if it needs to be written out. If no
            // I/O is in progress when try_to_write_page
            // returns, we can claim the page.

        Page := Page(CoreTop); // Consider frame's current resident.
        Frame := CoreTop; // And consider the frame.
        CoreTop := Next (CoreTop); // Make the page under consideration
        // the most recently used. If we ultimately
        // cannot claim it, this was the right move.
        // If we do not, it will be due to recent
        // use or I/O, and it is still good.
        call try_to_write_page (Page); // See if page needs writing out.
        if not lo_in_progress (Page) // If try_to_write_page succeeded in
            then do;
                call make_nonaccessible (Page); // Turn off access to Page. We do not
                    // exit this call until all processors
                    // have verified that they have flushed
                    // Descriptor (Page) from
                    // their associative
            end;

        call post_io_error;
        io_skip_counter := 0;
        CoreTop := Next (CoreTop);

    end;
try_to_write_page:
procedure (Page);

   dcl Top_of_pd_used_list PDrec;

   if On_pd (Page) then if Modified (Descriptor (Page)) then call write_page (Page,"modified","pd_ok"); // Page has been modified.
   else
      if Gtpd (Page) or in_use (Top_of_pd_used_list) then if Modified (Descriptor (Page)) then call write_page (Page,"modified","no_pd");
      else
         if Modified (Descriptor (Page)) then call write_page (Page,"modified","out_of_pd");
         else if Modified (Descriptor (Page)) then call write_page (Page,"modified","out_on_fd");
         else
            call write_page (Page,"not_mod", &pd_on_pd"); // Write even if not modified.
   return;
end try_to_write_page;
write_page

write_page: procedure (Page, Modflag, PDflag);

declare Modflag literal, PDflag literal;

if page_is_zero (Page) then return;

Modflag (Descriptor (Page)) := "false;"

call clear_associative_memory;

call_dramассей(Page) := Page;

call Page (Coreadd(Page)) := "write;"

if Devadd (Page) = "null;"

then Devadd (Page) := allocate_disk_record();

if PDflag = "put_on_pd"

then call allocate_pd(Page);

if on_be (Page)

then do;

if Modflag = "modified"

then Modflag_from_disk (PDrec (Page)) := true; // we must indicate that the pd copy that we

are about to start writing is different from the

disk copy.

call pd_thread_to_top (PDrec (Page)); // indicate that this pd record has seen recent use.

and

call device_write (Devadd (Page), Phys_Coreadd (Coreadd (Page))); // Start the actual write.

return;

end write_page;
page_is_zero:

FUNCTION P (Page);

// This procedure is called to determine whether
// or not a page frame contains all zeros. If it
// does, any disk or paging device space allocated to
// the page is relinquished.

IF (Phys_Coreda (Coreda (Page))) NOT (1024 words) = 00000000...

// See if page frame physically
// contains zeros.

THEN do:

call make_nonaccessible (Page);

// Turn off the addressability of the page. We will
// check again if page is all zeros since the page has
// not been addressable. We do this instead of simply
// turning off address checking because the vast
// majority of pages checked for zero are not zero, and
// turning off the access would cause another processor
// attempting to reference this page to fault, and loop on
// the page table lock, which we have elected.

IF (Phys_Coreda (Coreda (Page))) NOT (1024 words) = 00000000...

// See if still zero.

THEN do:

Modified (Descriptor (Page)) = false;

// Indicate that we are aware of page having been
// modified (i.e., zeros). We need not clear the associative
// memories, because, you will recall, access is off.

IF On_pd (Page)

THEN do:

Pevadd (Page) = Hiskadd (Pordc (Page)); // Page's address home is disk.

in_use [Pordc (Page)] = false;

// The pd record may be used.

On_pd (Page) = false;

// The page may be given.

Number_of_free_pd_records = Number_of_free_pd_records + 1;

// Increment free record count.

call pd_thread (s) (Pordc (Page));

// Suggest this

call alloc (Pordc (Page));

// pd record for immediate claiming
// by allocate pd.

end_if:

// Up to here, we have failed to page

call dealloc (Pordc (Page));

// Before the next step, the page
// is not in use. Actually, the
// page is in relinquish disk space.

call relinquish_disk_space (Pevadd (Page));

// From the disk block associated to Page

DEvoid (Page) = "null";

// Indicate that Page must be all
// zeros.

if (we were called from the page fault
then do:

// Page can be considered to have
// Page is zero.
sequence as opposed to post- // been written. This non-local goto
merge or other activity) // seems fine, since the effort of
then goto not_and_in find_core; // turning off access, which is already
return zero; // off, and unmasking the entire system
and // with the other global associative
and page_is_zero;
also will make accessible (Page);

// Page is not zero, return this page.
return zero; // was zero.
if (page > 0) { // if page already
if (page & 0) // was modified since first
write...
write to (Page)
while there was a reference to this
page and
write the zero
} // if the page is true we were unable to
write to it
else return 0;

if (page > 0) { // if page already
write to (Page)
while there was a reference to this
page and
write the zero
} // if the page is true we were unable to
write to it
get_free_pd_record

    declare Top_of_pd_used_list type PDrec;
    local Save_pd_first type PDrec;
    declare Number_of_free_pd_records arithmetic;
    local free_ptr arithmetic, PDrec;
    local First_time boolean;

    First_time := true;
    Save_pd_first := Top_of_pd_used_list;
    free_ptr := 0;

    do forever
      if First_time then
        free_ptr := Top_of_pd_used_list
        First_time := false
      else
        repeat Next_pd (PDrec)
          while (First_time or (PDrec = Save_pd_first));
      end if

      if in_use (PDrec) then
        if Free (PDrec) then
          call pd_thread_to_bottom (PDrec); // surely among the most recently used on the pd.
          if Modified_from_disk (PDrec) then
            if pd_used (PDrec) then
              call start_rws (PDrec); // we must free pd_used
              free_ptr := in_use (PDrec) + 1
            end if
          end if
        end if
      end if
    end do

    if free_ptr > 30 then failure; // should we have made enough pd records free
    return; // (see page fault)
post_page: procedure(Coreadd);  // This procedure is responsible for changing the
// state of page control data bases when the completion
// of an I/O operation is observed. It is invoked
// from individual device control routines.

local Page;

if Rws_in_frame (Coreadd)
  then call rws_done (Coreadd);
else do;
  Page := Page(Coreadd);
  lo_In_progress (Page) := false;
  if lo_read_or_write (Coreadd) = "read"
    then do;
      Coreadd(Page) := Coreadd;
      call make_accessible (Page);
    end;
  else do;
    writes_outstanding := writes_outstanding - 1;  // Maintain heuristic for find_core.
    call thread_to_top (Coreadd);  // Make this core frame the most likely
    // candidate for claiming. The usual reason that a
    // write was started is that it was a good candidate for
    // claiming in the first place. If Page has been used,
    // (this includes modified) since the Usage bit was turned
    // off, find_core will not claim this page now. Otherwise,
    // it will be the very next page claimed.
  end;
end;  // Cause any process waiting for the completion of this
call notify (Event(Page));  // I/O operation to resume.

return;
end post_page;
start_rws:

procedure (PDrec);
  declare Rws_active_count arithmetic;
  Coreadd := find_core();
  Rws_in_frame (Coreadd) := true;
  Rws_in_progress (PDrec) := true;
  io_read_or_write (Coreadd) := "read";
  Coreadd (PDrec) := Coreadd;
  PDrec(Coreadd) := PDrec;
  call pd_thread_out (PDrec);
  call device_read (Nevad (PDrec), Phys_Coreadd(Coreadd)); /* Start and possibly finish the read.
  call meter_disk (Disk_addr(PDrec), "write"); /* Meter motion to the disk.
  Rws_active_count := Rws_active_count + 1;
  Number_of_free_pd_records := Number_of_free_pd_records + 1; /* Indicate that this pd record
  Modified_from_disk(PDrec) := false;

  do while Rws_active_count > 30;
    call pass_any_io;
    end;
  end;

  return;
end start_rws;
rws_abort

rws_abort: procedure (Page);
local PDrec;

PDrec := PDrec (Page);
if Abort_flag (PDrec) then
   Abort_Complete (PDrec);
else
   Abort_flag (PDrec) := true;
   call make_accessible (Page);
   call meter_disk (Diskaddr (PDrec), "read");
   if this is not a record of the type to be
      taken out of the disk, and hence we
      require both the page record and a fetch
      of the page, we then become recently used.
   call pd_thread_to_bottom (Coreaddr (PDrec));
   make pd thread appear recently used.
   core (PDrec), in_use (PDrec) := true;
end; // Maintain this heuristic.
else Abort_flag (PDrec) := true;
   return;
end; // Return to page_fault with Rws_in_progress off.
and rws_abort;

// This procedure is invoked when a page fault is taken on a page which has
// an rws in progress.
// There are three such cases. 1) No abort has
// been initiated. We initiate one, and wait
// for notification from rws_done. 2) Another
// process has initiated one. We wait for it.
// 3) We have been notified by rws_done, and must
// clean up the abort.
// The PD record is of intense interest here.
// If this is 1 or 2 above.
// If this is so: Case 3. We clean up, and the
// RWS and the rws_abort are over.
// No more PD abort. Or TW.
// This rws_abort represents negation of
// page movement to disk, and hence we
// require both the page record and a fetch.
// pd thread to bottom (Coreaddr (PDrec)).
// We make pd thread appear recently used.
// Set core (PDrec), in_use (PDrec) := true.
// Return to page fault with Rws_in_progress off.
// Case 2. Abort already started. page_fault will
// wait for it.
// Case 1. Abort the rws. page_fault will wait for it.
// Return to page_fault to either continue
// and restart the fault, or wait.
rws_done: procedure(Coreaddr);

declare Rws_active_count arithmetic, Writes_outstanding arithmetic;
local PDrec;
PDrec := PDrec(Coreaddr);

if Abort_flag(PDrec)
    then do:
        if lo_read_or_write(Coreaddr) = "read"
            then Modified_from_disk(PDrec) := true;
        else Writes_outstanding := Writes_outstanding - 1;
    // Otherwise, maintain write count.
    Abort_Complete(PDrec) := true;
    // Indicate that we have aborted the rws. We
    // cannot make the page addressable because
    // (point of fact) we have no way to locate
    // Page(PDrec) in the actual implementation.
    // This is due to not having enough space to
    // save the required pointers.
    // We now cause all processes who faulted on
    // this page during the rws to resume. They
    // will all re-take the page faults which made them
    // first see this rws, and the first one to lock
    // the page table lock will make the page
    // addressable, the others will find that
    // there is no more page fault (see page_fault)
    // and simply return.
    call notify(Event(PDrec));
end;
else do:
    if lo_read_or_write(Coreaddr) = "read"
        // A normal rws read or write completion.
        // If a read finished, start the write.
        then do:
            lo_read_or_write(Coreaddr) := "write";
        // Indicate I/O direction for next time.
        call device_write(Diskaddr(PDrec),Phys_Coreaddr(Coreaddr));
        // Start the write.
        end;
    else do:
        // The write, and hence the rws, has finished, successfully
        // (i.e., without an abort.)
        Rws_active_count := Rws_active_count - 1;
        // Maintain the rws activity heuristic.
        Writes_outstanding := Writes_outstanding - 1;
        // Maintain find_core's heuristic.
        Rws_in_frame(Coreaddr) := false;
        // Turn off rws indicator.
        Rws_in_progress(PDrec) := false;
        // Turn off rws indicator for pd record.
        in_use(PDrec) := false;
        // This record is now free.

call pd_thread_to_top (PDrec);  // This record is now claimable.
Devadd (Page(PDrec)) := Diskaddr(PDrec);  // The page that was on this pd record
   // is now on disk.
On_nd(Page(PDrec)) := false;  // Let all know.
Page (Coreadd) := "null";  // The core block used as an rws buffer is now
   // immediately claimable.
call thread_to_top (Coreadd);  // Make it best candidate for claiming.

end;  
end rws_done;

return;
auxiliaries

/∗ Although some of these short routines might better be expressed in
line, they are conceptually modules in their own right, and
may be called from other points in the system. ∗/

/∗ Small auxiliary routines ∗/

device_read:procedure (Devadd, Phys_Coreadd);  // Called to initiate a read -
  // selects correct I/O routine.
    declare Phys_Coreadd arithmetic;
    if Device(Devadd) "= "drum"
      then call meter_disk (Devadd, "read");  // meter disk reads
        call (select_io_routine_entry(Device(Devadd), "read"))
        (Phys.Devadd (Devadd), Phys_Coreadd);  // call right routine.
    end device_read;

device_write: procedure (Devadd, Phys_Coreadd);  // Called to write a page.
    Writes.outstanding := Writes.outstanding +1;
    call (select_io_routine_entry(Device(Devadd), "write"))
      (Phys.Devadd (Devadd), Phys_Coreadd);
end device_write;

make_accessible: procedure(Page);  // Called to make a page accessible.
    Phys.Coreadd (Descriptor(Page)) := Phys_Coreadd(Coreadd(Page));  // Fill in physical address.
    Addressable (Descriptor(Page)) := true;  // make page addressable.
end make_accessible;

make_nonaccessible: procedure(Page);  // Called to make a page non-accessible.
    Addressable (Descriptor(Page)) := false;  // make page non-addressable.
    call clear_associative_memory;  // flush descriptor from associative memories.
end make_nonaccessible;

meter_disk: procedure (Devadd, Type);
    declare Type literal, Experiment_active boolean;
    if not Experiment_active then return;  // cannot accumulate data if buffer not wired.
    Trace_datum:=construct Trace_datum(Devadd:Devadd, Type:Type);
enqueue (Trace_datum, Trace_queue);
end meter_disk;

post_any_io: procedure;

// This routine is called in any situation where page
// control discovers some I/O bottleneck.
// It polls I/O routines for complete status. They will
// call post_page if any status arrives.

declare io_devices set literal;

local Device literal;

do Device := range io_devices;

    call (select_io_routine_entry (Device, "post"))()

end;

end post_any_io;
fixed_head_control

/* A Typical Paging I/O Control Routine */

/* This routine is the I/O control routine for the fixed-head disk. There exist routines
   almost identical to it for the moving-head disk and drum. The routine select_io routine_entry
   (not given here) is used to select the appropriate entry of the appropriate routine
   given the device identifier and the function to be performed. */

declare Phys_Devadd arithmetic, Phys_Coreadd arithmetic;
fixed_head_read: procedure (Phys_Devadd, Phys_Coreadd);
   call fixed_head_start (Phys_Devadd, Phys_Coreadd, "read"); // Call common queueing routine.
end;

fixed_head_write: procedure (Phys_Devadd, Phys_Coreadd);
   call fixed_head_start (Phys_Devadd, Phys_Coreadd, "write"); // Call common queueing routine.
end;

fixed_head_start: procedure (Phys_Devadd, Phys_Coreadd, Direction);
   declare Direction literal;
   call fixed_head_post;
   enqueue (construct io_program
      (Phys_Devadd, Phys_Devadd, Phys_Coreadd, Phys_Coreadd, Direction, Direction,
      Next: "null"), Fixed_Head_Channel_Queue);
   if (fixed head disk is not busy) then call start_io(Fixed_Head_Channel_Queue);
   // There is now work for the fixed head disk. Start it if it is idle.
end;

fixed_head_post: procedure;
   do lo_status := range (any complete I/O status); // Look at all new status.
   remove io program (lo_status) from Fixed_Head_Channel_Queue;
   call post page (Coreadd (lo_status)); // inform page control. See the declaration of lo_status.
end;
Implementation of the Hardcore Meters

In this appendix, we relate the exact identities of the measured events of Multics page control with which this experiment was concerned. This is necessary both to provide validity for what we have done, and to help others design similar techniques for other systems. It is assumed that the first appendix has been at least partially understood, perhaps with the overview sections fully understood.

We also discuss here the techniques used in implementing the Multics Supervisor interface for this experiment.

As should be clear from Chapter 2, we are interested in metering movement of pages in and out of the composite entity of core-drum. This "movement" in fact consists of copy creation and copy destruction. Movement "into" core drum consists of the creation of a page copy in core-drum where there previously was none, and movement "out of" core drum consists of the destruction of core or drum copies of a page, such that there is no copy in core-drum. We speak of this creation and deletion as movement because it is represented as movement of pages in an LRU stack.

We will now analyze the different types of motion in and out of core-drum. Pages come into core-drum either from the outside, i.e., disk, or by being created in core. Pages entering from disk can only do so as the result of a page fault to disk or a pre-paging from disk, so a call to "meter_disk" (see Appendix A) was installed in the i/o dispatching routine to record all reads from disk. Pages created in core never involve input/output. For the most part, these are pages which were never touched before, and would thus cause a page fault no matter how
large core-drum were. These page faults, however, involve neither
multiprogramming, i/o, nor idle time, and are thus of slightly less
interest in performance predictions than more general page faults. We
chose to ignore them. There is one other type of inward motion, which
will be motivated in our discussion of outward motion.

Outward motion consists of oustings from core-drum. This consists
of oustings from drum (which, as can be verified from "get_free_pd_record"
in the last appendix can only happen if there is no copy in core) or from
core. Oustings from core are only oustings from core-drum if page con-
trol (specifically, "try_to_write_page") decides that it should not be
written to the drum because of either lack of space there or the concerned
page is one of the special-cased "gtpd" pages. We will first consider
the oustings from drum. The ousting of a page which is different than
its disk copy, if one was ever made, is accomplished by the initiation of
a read-write sequence (rws). These rws initiations were thus metered as
outward movement. The ousting of a page which is identical to a disk
copy is done by simply claiming the drum frame (see "get_free_pd_record"),
and this event was likewise noted. Oustings from core interest us when-
ever they are not oustings to the drum. (We define an ousting "from core
to the drum" to be an ousting from core when a copy of the concerned page
is on drum. Note that this implies an ordering of the hierarchical
memory system.) These oustings from core normally happen only for the
special "Global transparent paging device (gtpd)" pages of the root direc-
tory, whose treatment was already fully covered, and in bad cases of page
faults or rws initiations, when there are no free drum frames available.
This case was also covered. As an interesting consequence of this definition of an ousting not to the drum, we observed a large number (approximately 2400 per hour) of oustings of pages which were all zeros, and were all zeros when brought into core (the conjunction of these statements essentially implies that these pages had no copies on either drum or disk).* Some special experiments designed to discover the source of this peculiar traffic were essentially fruitless. The data reduction programs described in section 2.3 were modified to ignore these anomalous oustings.

One consequence of metering read-write sequence initiation as outward motion is that the aborting, or reversal due to a page fault, of a read-write sequence must be metered as inward motion. This was done (see "rws_abort").

One remaining event which had to be metered was that of page destruction. The event we chose to represent this destruction was the handing back of the disk frame, if one existed, to the free disk pool, of any disk frame at all. This happens in two cases. First, explicit page destruction via the deletion of segments of the virtual memory requested by supervisor call, or their explicitly requested truncation causes this to happen. Secondly, as we have described, find_core deallocates both disk and drum frames when a page containing all zeros (a void page) is found with its used bit off. As described in section 2.2, we are interested only in the destruction of pages which are not in core-drum. The destruction of any such non-void page will always involve the deallocation of a disk frame, and thus will be properly metered. The destruction of void pages is not

*Even though this constituted about one quarter of all core-drum oustings, they bear absolutely no significance to the experiment.
a significant event, as they do not occupy a place in either the LRU or LRU extension stacks of section 2.1. The discovery of a newly void page by find_core also causes such an event to be recorded in the trace data as a page deletion. However, this page cannot be in the extension stack, because it was found by find_core because it was, in fact, in core. The data reduction programs were aware of these out-of-list deletions, and duly ignored them. The destruction of pages in core-drum which were never ousted is handled and ignored by this same mechanism.
Interface Details

The Multics hard-core interface for this experiment was designed to be a semi-permanent part of the Multics system, and thus have as little effect as possible on it when not in use. Thus, a page of the virtual memory was allocated for the circular buffer described in section 2.3 and its auxiliary data. When the experiment was enabled, via highly privileged supervisor primitive, this page was given a dedicated page frame and withdrawn from the pool of pageable core. This was necessary to insure that page control, when storing data in this buffer, would not take a page fault. Page control was also made to check a switch (the "enabled/disabled" switch) as to whether or not this had been done before attempting to reference the buffer. Another highly privileged supervisor primitive freed the page frame given to this buffer, resetting this switch before doing so.

The copying of data out of this buffer, via privileged supervisor entry point, ostensibly requires simply copying its contents into a user-specified area. However, it was an aim of the interface design to insure that the buffer would not change while the information was being copied. This could happen by either the processor not doing the copying taking a page fault, or the processor doing the copying taking a page fault referencing the user's area. Hence, to insure that no page control activity took place while this data was being copied, the data-gathering primitive had to lock the "page table lock" while doing this copying. This, in essence, prevents page faults from being processed, and cannot be done until any other process has unlocked this lock. The effect of this lock is to insure that only one processor is in page control at a time. When
one has the page table lock locked, one must not take a page fault, or infinite looping will result when that processor tries to lock the page table

lock to process it. What is more, the page fault handler is not

Thus, it was necessary to "wire" a dedicated page of the virtual

memory (allocate a dedicated core page frame and withdraw the latter from

the pool of pageable core) to copy the wired buffer into. Both pages

being wired (the buffer and the temporary copy page) ensured that no page

fault would take place during the copy. The contents of the copy page

could then be copied to the user-specified area after the page table lock

had been unlocked. The copy made grants each process area had been unlocked.

A further difficulty arose because the segment containing the tem-

porary copy page is a one-per-system segment, and thus could not be used

by two processes simultaneously. Thus, a lock had to be used to exclude

any other process from gathering data or making copies until the

such use of this segment. This lock would be locked by any process wanting
to gather data before it wired the temporary copy page, and unlocked after

it had been unwired. A process or finding the lock locked would be multi-

programmed, and the associated process notified when the lock was unlocked.

The code which copies the wired buffer into the temporarily wired

temporary copy page is entered only when the latter has been wired. How-

ever, it is possible that the former may not be wired, specifically, if

the experiment has not been enabled. If this is the case, a fatal page

fault with the page table lock locked would result. To avoid this, the

enabled/disabled switch must be checked by this code, but it cannot check

this switch until the page table lock is actually locked. Only when it is

locked can no page possibly be made unreferenceable, as no other process

can be in page control. The enabled/disabled switch is turned to disabled
BEFORE the buffer is unwired, and the buffer can only be made unreferenceable AFTER the page table lock is locked AFTER it has been wired. Thus, the sequence of events in a call to gather is as follows:

1. Attempt to lock the copy page lock; multiprogram and retry if failure.
2. Wire the temporary copy page.
3. Attempt to lock the page table lock; loop until successful.
4. Inspect the enabled/disabled switch; copy the wired buffer into the copy page if enabled, else copy zeros.
5. Unlock the page table lock.
6. Copy the temporary copy page out to the user-specified area.
7. Zero the temporary copy page, and unwire it.
8. Unlock the copy page lock; notify any waiting processes.

The step of zeroing the copy page is done so that this page will be immediately claimable to find_core. This is done as both a friendly gesture and an attempt to keep this page off of the drum and out of the disk traffic visible to the experiment. The page frame is always void when unwired (returned to the pool of pageable core).

The sequence for enabling the experiment is as follows:

1. Wire the buffer page.
2. Set the enabled/disabled switch to enabled.

The sequence for disabling the experiment is as follows:

1. Set the enabled/disabled switch to disabled.
2. Unwire the buffer page.

The only remaining question of locking is that of the buffer becoming unwired as page control is placing data in it. This cannot happen. Any page control operation sequence other than those just described can be summarized as:
1. Attempt to lock the page table lock; loop until successful.
2. Do all nature of page control.
3. Conditionally, unlock the page table lock and end this sequence.
4. Check the enabled/disabled switch; add data to the buffer if and
   only if it is enabled.
5. Go back to step 2.

The making unreferenceable of pages by find_core falls under step 2
above. During the checking of the switch and the placing of data in the
buffer, this making unreferenceable cannot happen on this processor. The
page table lock excludes any other processor, and there is no problem.
Appendix C

System Performance Graphs during Experiments

We present here graphs of user load, cpu utilization, and paging over-
head as functions of time of day on the days of the two experiments, "dtm
21" and "dtm 23". This data was condensed from a graphical presentation
of these parameters routinely prepared by the M.I.T. Information Processing
Center. It is given here to provide a feeling for the relative user load
during the experiment, and to allow a rough approximation to total system
headway during the experiment to be computed. This may be computed by
multiplying the time of the experiment (roughly 14 hours, or 50,000 seconds)
by the fraction of the system which was not idle time or paging overhead
(quite roughly, 40%), obtaining 20,000 seconds, and multiplying by the
system memory reference rate (400,000 references per second), obtaining
$8 \times 10^9$ virtual memory references.
Figure C.1 User Load during Experiments

- --- dtm 21, 0945 - 2359
  May 7, 1973

- ---- dtm 23, 0945 - 2359
  May 14, 1973

User load, Users

Time of day, EDT
Figure C.2  Percent of Total System Time Spent Idle, as a Function of Time of Day (approximate). All types of idle time combined.

- - - - - - dtm 21
- - - - - - dtm 23

% of total system time spent idle

Time of day, EDT

1000  1200  1400  1600  1800  2000  2200  0000
Next day
Figure C.3  Percentage of Total System Time
Spent in Paging Overhead as a Function of Time (approximate)

--- dtm 21

----- dtm 23

Time of day, EDT


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**Author(s)**

Bernard S. Greenberg

**Performing Organization Name and Address**

PROJECT MAC; MASSACHUSETTS INSTITUTE OF TECHNOLOGY:
545 Technology Square, Cambridge, Massachusetts 02139

**Sponsoring Organization Name and Address**

Office of Naval Research
Department of the Navy
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**Abstract**

This thesis reports the design, conducting, and results of an experiment intended to measure the paging rate of a virtual memory computer system as a function of paging memory size. This experiment, conducted on the Multics computer system at M.I.T., a large interactive computer utility serving an academic community, sought to predict paging rates for paging memory sizes larger than the existent memory at the time. A trace of all secondary memory references for two days was accumulated, and simulation techniques applicable to "stack" type paging algorithms (of which the least-recently-used discipline used by Multics is one) were applied to it.

A technique for interfacing such an experiment to an operative computer utility in such a way that adequate data can be gathered reliably and without degrading system performance is described. Issues of dynamic page deletion and creation are dealt with, apparently for the first reported time. The successful performance of this experiment adds the viability of performing this type of measurement on this type of system. The results of the experiment are given, which suggest models of demand paging behavior.

**Key Words and Document Analysis**

**Descriptors**

Virtual memory
Demand paging
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**Identifiers/Open-Ended Terms**

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