Wait-Free Regular Storage from Byzantine Components

Ittai Abraham* Gregory Chockler† Idit Keidar‡ Dahlia Malkhi*

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Abstract

We present a simple, efficient, and self-contained construction of a wait-free regular register from Byzantine storage components. Our construction utilizes a novel building block, called 1-regular register, which can be implemented from Byzantine fault-prone components with the same round complexity as a safe register, and with only a slight increase in storage space.

Keywords: shared-memory emulations, Byzantine failures, wait freedom.

*School of Computer Science and Engineering, The Hebrew University of Jerusalem. Email: {ittaia,dalia}@cs.huji.ac.il.

†Lab for Computer Science and Artificial Intelligence. Massachusetts Institute of Technology. Email: grishac@theory.lcs.mit.edu.

‡Department of Electrical Engineering, The Technion – Israel Institute of Technology. Email: idish@ee.technion.ac.il.

1 Introduction

In this paper we consider a problem of constructing wait-free distributed storage from Byzantine fault prone components in asynchronous settings. Our specific objective is to devise a solution that will be simple, efficient, and feasible in practice, yet providing meaningful semantics for higher level applications. Roughly speaking, a wait-free object is one that always guarantees the liveness of shared memory operations, including in the presence of any number of process (client) failures.

Constructing efficient storage solutions from Byzantine components received considerable attention recently, as such solutions are useful in a number of emerging application domains. Originally, such solutions have been introduced in the context of scalable client-server systems. Such systems achieve more scalability than traditional state-machine replication approaches by removing the direct communication among the servers, and thus reducing the load that each client request (or transaction) imposes on the servers. This approach was pioneered in the Fleet [17] system, and adopted in many others, e.g., SBQ-L [18], Agile Store [11], Coca [28], and [5]. Similar solutions have also been employed in the setting of Storage Area Networks (SANs). SAN technology allows clients to access disks directly over the network so that the file server bottleneck is eliminated. Examples of SAN-based systems that use disks for information sharing and coordination include Compaq's Petal [13] and Frangipani [23], Disk Paxos [7], Active Disk Paxos [6], and Byzantine Disk Paxos [1]. More recently, solutions of this nature have been adopted in peer-to-peer systems, which consist of a collection of widely spread nodes storing data objects. Naturally, due to their Internet-wide deployment, the storage nodes are prone to malicious attacks, which motivates adopting a Byzantine failure model for the storage nodes. Examples of peer-to-peer systems that adopt storage-centric replication to support availability in face of Byzantine failures include Rosebud [21] and [14].

Fault-prone storage systems as mentioned above can be formally modeled as an asynchronous shared memory system where a threshold t of the memory objects may fail by being non-responsive [3, 10] or by returning arbitrary values [2, 10] (i.e., by being Byzantine); this failure model is called *non-responsive arbitrary (NR-Arbitrary) faults* [10]. In this paper, we assume that less than one fourth of the memory objects can fail. In [1] we show that this assumption is necessary for achieving wait-free implementations as efficient as those presented in this paper; that is, every construction that uses less than $4t + 1$ objects must have a higher latency.

All existing wait-free Byzantine-resilient storage constructions provide *safe* register semantics[1] [10, 16, 1]. The only previous direct constructions of objects with stronger (regular or atomic) semantics from Byzantine storage satisfy weaker (non-wait-free) termination conditions [18, 1]. In this paper we focus on wait-free constructions. Safe register semantics, by themselves, are too weak to be directly useful for applications. The focus on these semantics has been justified by the existence of known reductions from wait-free safe registers to stronger ones [19, 27, 8, 12, 24, 25, 26, 9]. However, this approach results in constructions that are not self-contained, and as we argue below, are not tailored to the requirements of a distributed storage system. In this paper, we do not use these reductions as black boxes, but instead capitalize on their techniques in order to derive a new self-contained wait-free regular register construction that is simple, efficient, and feasible in distributed storage environments.

Most existing constructions of strong memory objects are fairly elaborate. We believe that a

[1]A safe register guarantees that every *read* operation that does not overlap any *write* returns the latest written value, or the initial value if no value was written; the result of a *read* operation that does overlap a *write* operation may be arbitrary.

major reason for this complexity is the fact that they aim to achieve an atomic register. However, recent studies indicate that storage with *regular* semantics is sufficient in most cases [7, 22, 1]. A regular register is weaker than an atomic one; roughly speaking, a *regular register* guarantees that every *read* operation returns the value that was written by a *write* operation invoked not earlier than the last *write* operation that returns before the *read* is invoked, or the initial value if no value is written before the *read*. We therefore focus on constructing regular registers in this paper.

Existing constructions of strong (regular or atomic) wait-free objects from weaker ones were not designed with distributed storage in mind. In particular, such constructions have typically focused on bounding the memory size rather than reducing the number of shared memory accesses. In a distributed setting, however, every memory access incurs a latency of two message delays, whereas storage space is typically abundant. Therefore, we believe that a practical construction for the model we consider herein should focus on simplicity and reducing communication costs, even at the cost of using unbounded counters. This is precisely the approach we take in this paper. We give an algorithm that uses unbounded counters, (which is acceptable in practice), achieves better latency than all existing regular register implementations, and is very simple to understand and implement.

Our approach further differs from existing constructions in the basic building blocks employed. Traditional shared memory constructions often use a collection of safe single-bit registers, which are mathematical models of flip-flops. In contrast, in a distributed storage setting, we can assume that each storage unit (representing a disk or a server) can support stronger objects. In this paper we introduce a novel intermediate building block, called 1-*regular* register. We show that a 1-regular register can be implemented from Byzantine fault-prone components with the same round complexity as a safe register, and with only a slight increase in storage space. We then give a simple and efficient implementation of a wait-free regular register using 1-regular ones.

Outline: The rest of this paper is organized as follows: In Section 3, we describe the formal computation model used throughout the paper and give the definitions of various register types. In Section 4, we construct a wait-free 1-regular register from $n > 4t$ base registers up to t of which can incur Byzantine faults. Finally, in Section 5, we show how to use 1-regular registers in order to construct a wait-free regular register. We note that all but one of the 1-regular registers employed in this construction can be replaced with (weaker) safe ones. Therefore, for completeness, we present a direct safe register construction in the appendix. As noted above, using safe registers instead of 1-regular ones does not reduce the latency, but it slightly reduces the space requirements.

2 Related Work

Wait-free shared register constructions have been an actively researched area for several decades [19, 27, 8, 12, 24, 25, 26, 9]. Most of the constructions found in the literature aim to implement atomic registers from safe bits. One notable exception is the construction by Lamport in [12], which implements an n-valued regular register using $O(n)$ safe bits. The construction of [27] was purported to be an atomic register construction, but in fact, only provides regular semantics.

Peterson [19] and Tromp [24] present constructions of atomic registers from safe bit tracks and atomic control bits. It appears that these constructions can be easily adapted to implement regular registers by replacing the atomic control bits with regular ones, which in turn can be easily obtained from safe bits as shown in [12]; nevertheless, this was neither claimed nor proven in those papers. Although both of these constructions have logarithmic space complexity, the number of shared memory accesses they employ is rather high. Therefore, they are not directly applicable in a distributed setting. Nonetheless, our work benefits from several techniques and ideas underlying these constructions, e.g., using separate tracks to store copies of the register value, and using a handshake mechanism to coordinate between the reader and the writer.

All the existing wait-free Byzantine-fault-tolerant register constructions for distributed storage setting only provide safe semantics [16, 10, 1]. Other constructions achieve stronger semantics at the cost of weaker termination guarantees: the protocols by [18] and [4] implement atomic and regular registers, respectively, but do not guarantee termination in the face of client crashes; and the algorithm of [1] implements a regular register where the read operation is guaranteed to terminate only if it eventually runs in isolation for sufficiently long.

Our 1-regular register notion is a generalization of the pseudo-regular register of [20]. Whereas a read operation on our 1-regular register can return $\perp$ if it is concurrent with more than one write, a read from the pseudo-regular register is allowed to return $\perp$ if it is concurrent with any number of writes. Hence, the guarantees it provides corresponds to 0-regularity in our terminology.

3 The System Model

We consider asynchronous shared memory systems consisting of a collection of processes interacting with a finite collection of objects. Objects and processes are modeled as I/O automata [15]; for space constraints, we do not repeat the details of the I/O model here.

An object automaton's interface is determined by its *type*, which is a tuple consisting of the following components: (1) a set $Vals$ of values; (2) a set of *invocations*; (3) a set of *responses*; and (4) a *sequential specification*, which is a function from *invocations* $\times$ $Vals$ to *responses* $\times$ $Vals$. In a shared memory system consisting of processes $P_1, P_2, \ldots$, an object of type $\mathcal{T}$ interacts with a process P_i by means of input actions of the form a_i, where a is an invocation of $\mathcal{T}$, and output actions of the form b_i, where b is a response of $\mathcal{T}$. An object's external behavior is specified in terms of the properties of its traces (i.e., executions consisting of external actions only). Liveness properties are required to hold only in *fair executions*, i.e., executions where each output and internal action has infinitely many opportunities to occur.

The interaction between a process and an object is *well-formed* if it consists of alternating invocations and responses, starting from an invocation. We only consider systems where interactions between processes and objects are well-formed. Well-formedness allows an invocation occurring in an execution α to be paired with a unique response (when such exist). If an invocation has a response in α, the invocation is *complete*; otherwise, it is *incomplete*. Note that well-formedness does not rule out concurrent operation invocations on the same object by different processes. Nor does it rule out parallel invocations by the same process on different objects, which can be performed in separate threads of control.

A threshold t of the objects may suffer NR-Arbitrary failures [10], i.e., may fail to respond to an invocation, or may respond with an arbitrary value. Any number of the processes may fail by stopping.

3.1 Registers

A *read/write register* (or simply, register) type supports an arbitrary set $Vals$ of values with an arbitrary initial value $v_0 \in Vals$. Its invocations are *read* and *write*(v), $v \in Vals$. Its responses are $v \in Vals$ and *ack*. Its sequential specification, f, requires that every *write* overwrites the last value written and returns *ack* (i.e., $f(\mathit{write}(v), w) = (\mathit{ack}, w)$); and every *read* returns the last value written (i.e., $f(\mathit{read}, v) = (v, v)$). In a shared memory system consisting of processes $P_1, P_2, \ldots$, a process P_i interacts with a shared register by means of input actions of the form read_i and $\mathit{write}(v)_i$, and output actions of the form v_i and ack_i. A read/write register is called *k-reader/m-writer* if only k (m) processes are allowed to read (resp. write) the register. We use the term multi-reader when the particular number of readers is not important.

We now define several register properties that will be used throughout the paper. Fix x to be a single-writer/multi-reader (SWMR) or single-writer/single-reader (SWSR) register, and let σ be a sequence of invocations and responses of x.

Safe register. σ is *safe* [12] if every complete *read* operation that does not overlap any *write* operation returns the register's value when *read* was invoked (i.e., the latest written value or the initial value v_0 if no value was written). A register is called *safe* if it has only safe traces.

Regular register. σ is *regular* [12] if it is safe, and in addition, a *read* operation that does overlap some *write* operations returns either one of the values written by overlapping *write*s or the register's value before the first overlapping *write* is invoked. A register is *regular* if it has only regular traces.

1-regular register. σ is *regular* if it is safe, and in addition, a *read* operation that overlaps at most one *write* operation returns either the value written by overlapping *write* or the register's value before the overlapping *write* is invoked. Otherwise, a read operation may return in addition a special $\perp$ value. A register is 1-*regular* if it has only 1-regular traces.

Wait Freedom. Register x is *wait-free* if in any fair execution of any shared memory system that includes x, every invocation of x by a correct process is complete.

4 Wait-free 1-Regular Register Construction

The implementation of a wait-free SWMR 1-regular register from $n > 4t$ wait-free SWMR regular registers up to t of which can incur NR-arbitrary failures is depicted in Figure 1. The notation INVOKE *write*(x_i, v), (respectively, INVOKE $tmp \leftarrow \mathit{read}(x_i,))$ means that a new thread is started that performs a *write* on register x_i with value v (respectively, a *read* of register x_i whose response will be stored in local variable *tmp*). The notation x_i RESPONDED means that the last thread created by an INVOKE operation has completed its execution on register x_i. Note that this is well defined because we maintain well formedness using control flags *pending* and *enabled* in such a manner that there is at most one pending thread for each register. Each of the n base registers x_i consists of a cyclic two-value buffer whose elements are denoted $x_i[0]$ and $x_i[1]$ respectively. Each WRITE operation WRITE(v) first chooses a unique monotonically increasing timestamp ts and then writes the pair $\langle ts, v\rangle$ to the base registers x_i provided that the kth WRITE operation updates the k mod 2th part of x_i. This mechanism ensures that every READ operation that is overlapped by at most one WRITE operation will always be able to recover a good value (i.e., the value that upholds regularity) from the values read from either part of the registers as explained below.

The READ implementation reads the values from at least $n-t$ base registers and stores the values read from the $x_i[0]$ and $x_i[1]$ components in the arrays $w0[1 \dots n]$ and $w1[1 \dots n]$ respectively. The predicate $\mathsf{safe}(c, w)$, where $c \in TSVals$ (i.e., a timestamp-value pair) and w is a vector of $TSvals$, evaluates to true if c appears in at least $t+1$ elements of w. A value c which is safe in either $w0$ or $w1$ is known to be returned by at least one correct register x_i and therefore, was written by some previously invoked WRITE. However, the safe predicate by itself is insufficient to ensure regularity since some old values could still be returned by $t+1$ registers (e.g., if there are t registers which are not up to date and 1 which is faulty). Hence, in order for a safe value c to be returned, we must ensure that all values with a timestamp higher than $c.ts$ are *invalid*, i.e., could not have been written by a later complete WRITE operation.

The validity check is accomplished by the predicate $\mathsf{invalid}(c, w)$, which returns true if and only if for at least $2t+1$ elements $w[j]$, $w[j].ts < c.ts$ or $w[j].ts = ts \wedge w[j].val \neq c.val$. Since every completely written value can only be overwritten by a higher timestamped value, a value c is invalid in $w0$ (resp. $w1$) if and only if it was either not written to at least $n-t$ components $x_i[0]$ (resp. $x_i[1]$), or was not written at all. Hence, every value c which is both safe in $w0$ (resp. $w1$) and for which all the higher timestamped values are invalid, is known to be not older than the value written by the last WRITE operation that writes $x_i[0]$ (resp. $x_i[1]$) and completes before READ is invoked. Therefore, the highest timestamped such value is guaranteed to preserve regularity.

The informal discussion above is formalized in the following two lemmas:

Lemma 1. *If* READ *returns $v \neq \perp$, then v is either a value that was written by an overlapping* WRITE*, or the register's value before the first overlapping* WRITE *was invoked.*

Proof. Let R be a READ operation that completes. Let v be the value written by the last WRITE operation W that completes before R is invoked. By the WRITE implementation, there exists a timestamp ts such that $\langle ts, v\rangle$ is written to either $x_i[0]$ or $x_i[1]$ component of at least $n-t$ base registers x_i. Hence, w.l.o.g. we can assume that $\langle ts, v\rangle$ was written to either one of two components, say to $x_i[0]$. Since READ returns a value $\neq \perp$, by the READ code, $C0 \cup C1 \neq \emptyset$. There are two cases to consider:

First, suppose that $C0 = \emptyset$ and $C1 \neq \emptyset$. By line 5, this implies that either (1) $\neg\mathsf{safe}(c, w0)$ for all $c \in TSVals$, or (2) for every $c \in TSVals$ such that $\mathsf{safe}(c, w0)$ holds, $\mathsf{higherValid}(c, w0)$ is also true. In both these cases, READ overlaps at least one complete WRITE operation that writes $\langle ts', v'\rangle$ with $ts' > ts$ to the $x_i[1]$ component of the base registers x_i. Hence, there are at most $2t$ base registers x_i such that $x_i[1].ts < ts$ or $x_i[1].ts = ts \wedge x_i[1].v \neq v$. Thus, for any $c' \in TSVals$ such that $c'.ts < ts$, $\mathsf{higherValid}(c', w1)$ is true. Therefore, for each $c1 \in C1$, $c1$ was returned by at least one correct register ($\mathsf{safe}(c1)$), and $c1.ts \geq ts$. Hence, $c1.val$ was written by a WRITE operation that follows W. Since $C1 \neq \emptyset$, the return value val is equal to $c1.val$ for some $c1 \in C1$. Hence, the regularity is maintained.

Finally, suppose that $C0 \neq \emptyset$. In this case, there are at most $2t$ base registers x_i such that $x_i[0].ts < ts$ or $x_i[0].ts = ts \wedge x_i[0].v \neq v$. Thus, for any $c' \in TSVals$ such that $c'.ts < ts$, $\mathsf{higherValid}(c', w0)$ is true. Since $C0 \neq \emptyset$, then for each $c0 \in C0$, $c0$ was returned by at least one correct register ($\mathsf{safe}(c0)$), and $c0.ts \geq ts$. Furthermore, by line 8, the value returned must have been written with a timestamp which is at least as high as $c0.ts$. Hence, the return value upholds regularity. □

Types: $TSVals = TS \times Vals$, with selectors ts, val;
Shared objects: regular registers $x_i \in TSVals \times TSVals$, $1 \le i \le n$; whose components are addressed by $x_i[0]$ and $x_i[1]$; initially $x_i = \langle\langle ts_0, v_0\rangle, \langle ts_0, v_0\rangle\rangle$;

WRITE Emulation

Local variables:
  $enabled[1 \ldots n],\ pending[1 \ldots n] \in$ Boolean
    initially $\forall i\ enabled[i] = pending[i] = false$;
  $w[2] \in TSVals$, initially $w[0] = w[1] = \langle ts_0, v_0\rangle$;
  $turn \in \{0, 1\}$, initially 0;
  $ts \in TS$;

WRITE(v):
  choose $ts \in TS$ larger than previously used;
  $w[turn] \leftarrow \langle ts, v\rangle$;
  for $1 \le i \le n,\ enabled[i] \leftarrow true$;
  repeat
    CHECK;
  until $|\{i : \neg enabled[i] \wedge \neg pending[i]\}| \ge n - t$;
  $turn \leftarrow \neg turn$;
  return ack;

CHECK:
  if ($\exists i : enabled[i] \wedge \neg pending[i]$) **then**
    $\langle enabled[i], pending[i]\rangle \leftarrow \langle false, true\rangle$;
    INVOKE $write(x_i, \langle w[0], w[1]\rangle)$;
  if ($\exists i : x_i$ RESPONDED) **then**
    $pending[i] \leftarrow false$;

READ Emulation

Local variables:
  $enabled[1 \ldots n],\ pending[1 \ldots n],\ old[1 \ldots n] \in$ Boolean
    initially $\forall i\ enabled[i] = pending[i] = false$;
  $w0[1 \ldots n], w1[1 \ldots n], tmp0[1 \ldots n], tmp1[1 \ldots n] \in TSVals$;

Predicate and macro definitions:
  $\mathsf{invalid}(\langle ts, v\rangle, w) \triangleq |\{i : w[i] = \langle ts', v'\rangle \wedge$
    $(ts' < ts \vee (ts' = ts \wedge v \neq v'))\}| \ge 2t + 1$;
  $\mathsf{safe}(c, w) \triangleq |\{i : w[i] = c\}| \ge t + 1$;
  $\mathsf{higherValid}(c, w) \triangleq \exists i : w[i] = c' \wedge c'.ts \ge c.ts \wedge \neg\mathsf{invalid}(c', w)$;

READ:
  for $1 \le i \le n$, **if**($pending[i]$) **then** $old[i] \leftarrow true$;
  for $1 \le i \le n,\ enabled[i] \leftarrow true$;
  for $1 \le i \le n$: $w0[i] \leftarrow \bot$, $w1[i] \leftarrow \bot$;
  repeat
    CHECK;
  until $|\{i : \neg enabled[i] \wedge \neg pending[i]\}| \ge n - t$;
  $C0 \leftarrow \{c0 \in TSVals : \mathsf{safe}(c0, w0) \wedge \neg\mathsf{higherValid}(c0, w0)\}$;
  $C1 \leftarrow \{c1 \in TSVals : \mathsf{safe}(c1, w1) \wedge \neg\mathsf{higherValid}(c1, w1)\}$;
  if ($C0 \cup C1 \neq \emptyset$) **then**
    return $c.val$: $c.ts = max\{c'.ts : c' \in C0 \cup C1\}$;
  return $\bot$;

CHECK:
  if ($\exists i : enabled[i] \wedge \neg pending[i]$) **then**
    $\langle enabled[i], pending[i]\rangle \leftarrow \langle false, true\rangle$;
    INVOKE $\langle tmp0[i], tmp1[i]\rangle \leftarrow read(x_i)$;
  if ($\exists i : x_i$ RESPONDED) **then**
    if ($\neg old[i]$) **then**
      $\langle w0[i], w1[i]\rangle \leftarrow \langle tmp0[i], tmp1[i]\rangle$;
    $pending[i] \leftarrow false$; $old[i] \leftarrow false$;

Figure 1: The 1-regular register emulation.

Lemma 2. *If $R =$ READ returns $\bot$, then R is concurrent with at least two WRITEoperations.*

Proof. Let $W =$ WRITE(v) be the last WRITE operation that completes before R is invoked, and ts be the timestamp used to write v to the base registers. Suppose w.l.o.g. that $\langle ts, v\rangle$ was written to the $x_i[0]$ component of the base registers x_i. Assume by contradiction that READ overlaps at most one WRITE operation $W' =$ WRITE(v') and let ts' be the timestamp used to write v' to the base registers. Since the base registers updated by W and W' intersect the base registers read by R by at least $t + 1$ correct registers, either $\mathsf{safe}(\langle ts, v\rangle, w0)$ or $\mathsf{safe}(\langle ts', v'\rangle, w1)$ (or both) are true. Moreover, there are at most $2t$ registers x_i such that $x_i[0].ts < ts \vee (x_i[0].ts = ts \wedge x_i[0].val \neq v)$ or $x_i[1].ts < ts' \vee (x_i[1].ts = ts' \wedge x_i[1]x.val \neq v')$. Hence, either $\neg\mathsf{higherValid}(\langle ts, v\rangle, w0)$ or

$\neg\mathsf{higherValid}(\langle ts', v'\rangle, w1)$ is true. Therefore, $C0 \cup C1 \neq \emptyset$ so that $\perp$ cannot be returned. A contradiction. □

The two lemmas above imply the following

Lemma 3 (1-Regularity). *All the traces of the algorithm in Figure 1 are* 1*-regular.*

Finally, both WRITE and READ are wait-free because neither one ever awaits more than $n - t$ replies and at least $n - t$ base registers are responsive. We proved the following

Theorem 1. *The algorithm in Figure 1 is an implementation of a wait-free* 1*-regular register from* $n > 4t$ *regular base registers at most* t *of which can incur NR-arbitrary faults.*

5 Wait-free Regular Register Construction

We now present a regular register construction from 1-regular and safe ones. We note that only one of the registers in this construction needs to be 1-regular; it suffices for the remaining regulars to satisfy the weaker safe semantics. For completeness, we show in Appendix A a direct construction of a wait-free safe register from $n > 4t$ regular base registers up to t of which can be Byzantine faulty, using a technique similar to that of [16].

The algorithm in Figure 2 uses one wait-free 1-writer/m-reader 1-regular registers, m 1-reader/1-writer safe registers writeable by the writer, and m 1-reader/1-writer safe registers writeable by the reader to construct a wait-free 1-writer/m-reader regular register.

The read and write operations to the shared safe and 1-regular registers are denoted *read* and *write*. We construct WRITE and READ operations that maintain regularity.

The two writer's registers are called P and B for primary and backup respectively. These registers are used as buffers (or *tracks*) to store the values written by the writer. Each reader process i, $1 \leq i \leq m$ uses a 1-writer/1-reader multi-valued safe register RL_i to signal the writer about a possible concurrent read in progress.

The construction works as follows: The writer starts by writing the primary register P (line 1). It then examines the reader registers RL_i to see whether the value of some of these registers has changed since the last time it was read by the writer. If so, it saves the last written value in the backup registers B_j for every register RL_j where a change was observed (lines 4–6) before returning *ack* (line 7).

The reader i starts by writing the register RL_i (line 2) and then proceeds to read the primary track P (line 3). We consider the following two cases:

1. The read of the primary track by the reader (line 3) returns a value $\neq \perp$. In this case, 1-regularity of P ensures that the return value preserves regularity;

2. The read of the primary track by the reader (line 3) returns $\perp$. In this case, 1-regularity implies that *read*(P) by i is concurrent with at least two writes to P by the writer. Therefore, the writer's code is executed in whole at least once since *read*(P) has been invoked. This implies that the writer observes the change in the value of the register RL_i, and therefore, writes the i's backup track exactly one time before *read*(P) completes. Therefore, when reader i eventually returns from *read*(P), it finds B_i already written, and therefore, returns a correct value.

Shared objects:

1-writer/1-reader safe registers $RL_i \in$ Integers, writeable by the reader i, $1 \le i \le m$, and readable by the writer; initially 0;
1-writer/m-reader 1-regular register $P \in Vals \cup \perp$ writeable by the writer and readable by the readers; initially v_0;
1-writer/1-reader safe register $B_i \in Vals$ writeable by the writer and readable by the reader i; initially v_0;

Local to the writer:
  Static $wl[1 \ldots m] \in$ Integers
    initially $\forall i\ wl[i] = 0$;
  $a_i \in$ Integers, for $1 \le i \le m$;

WRITE(v):
1: $write(P, v)$;
2: $a_i \leftarrow read(RL_i)$, **for each** i, $1 \le i \le m$;
3: **if** $(\exists i : a_i \neq wl[i])$ **then**
4:   **for each** i, $a_i \neq wl[i]$:
5:     $wl[i] \leftarrow a_i$;
6:     $write(B_i, v)$;
7: **return** ack;

Local to the reader i, $1 \le i \le m$:
  Static $rl_i \in$ Integers, initially 0;
  $x \in Vals \cup \{\perp\}$;

READ$_i$, $1 \le i \le m$:
1: $rl_i \leftarrow rl_i + 1$;
2: $write(RL_i, rl_i)$;
3: $x \leftarrow read(P)$;
4: **if** $(x = \perp)$ **then**
5:   $x \leftarrow read(B_i)$;
6: **return** x;

Figure 2: The 1-writer/m-reader Wait-Free Regular Register Construction.

We observe that all the writes to the backup registers (line 6) can be executed in parallel. Hence in a distributed implementation, updating all B_i's would incur only the single round-trip message latency. This could be optimized even further by combining the writes targeted to the same destinations within a single message.

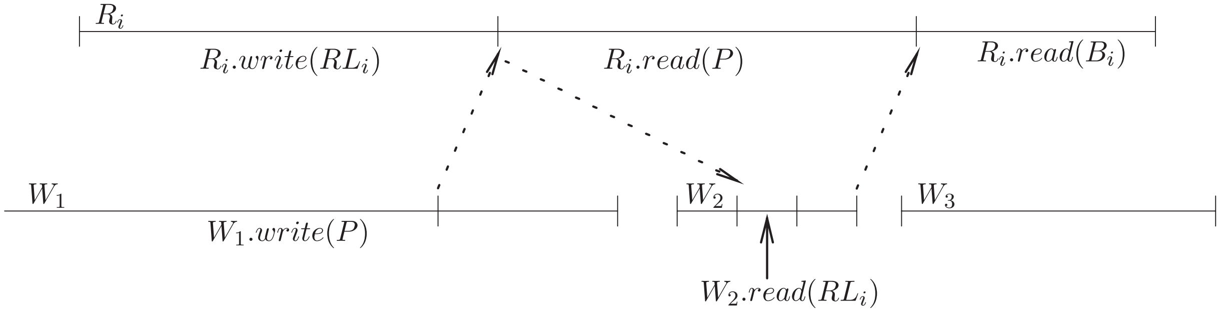

Figure 3: Several WRITE operations overlapping READ.

The above intuition is formalized by the following lemma:

Lemma 4. *Let* READ *overlap one or more* WRITE *operations. Then* READ *returns one of the values written in an overlapping* WRITE*, or the value written in the latest* WRITE *operation preceding the* READ.

Proof. (We refer the reader to Figure 3 for intuition on this proof.)

Let R_i be a READ operation by process i. Let R_i consist of a write $R_i.write(RL_i)$ to RL_i; a read $R_i.read(P)$ of P; and potentially a read $R_i.read(B_i)$ of B_i.

For any WRITE operation W, we refer to specific operations within WRITE as follows. We denote by $W.write(P)$ the first write operation (to P). We denote by $W.read(RL_i)$ the read from RL_i, $W.a_i$ the value returned from $W.read(RL_i)$, and $W.wl$ the value of wl at the beginning of W. We let $W.write(B_i)$ denote the write to B_i, if exists.

Let W_1 denote the latest WRITE such that $W_1.write(P)$ terminates before $R_i.Write(RL_i)$ completes. Let the two WRITE operations succeeding W_1 be denoted W_2 and W_3, respectively.

By choice of $W_1.write(P)$, $W_2.read(RL_i)$ strictly succeeds $R_i.write(RL_i)$ (see Figure 2). Hence, if no WRITE operation before W_2 sees the value written in $R_i.write(RL_i)$, then W_2 must have $W_2.wl \neq W_2.a_i$, and $W_2.write(B_i)$ exists. Otherwise, some WRITE preceding W_2 already sees $R_i.write(RL_i)$ and writes B_i. In any case, at the latest when W_2 completes, a write to B_i completes. Moreover, B_i is written at most once during $R_i.write(RL_i)$. The value stored in B_i in this write is the most up-to-date value written in a WRITE preceding or concurrent with R_i.

Now there are two possible cases. The first case is when $R_i.read(P)$ returns a value other than $\perp$. Then by 1-regularity, it returns a good value from P (i.e., it returns a non-$\perp$ value of an overlapping WRITE or the latest WRITE preceding the read).

The second case is when $R_i.read(P)$ returns $\perp$. Then by 1-regularity, $R_i.read(P)$ overlaps two WRITE operations. Since W_1 strictly precedes $R_i.read(P)$, we have that W_2 and W_3 overlap $R_i.read(P)$. Hence, W_2 completes before $R_i.read(P)$ terminates. As we show above, at the latest, W_2 is a WRITE that sees $R_i.write(RL_i)$ and updates B_i. Since $R_i.write(RL_i)$ completes before $R_i.read(B_i)$ starts, and because no new *write* to B_i is invoked until $R_i.read(B_i)$ returns, by safety of B_i, $R_i.read(B_i)$ returns a non-$\perp$ value that upholds regularity. $\square$

Finally, if a READ operation R_i by a process i is not concurrent with any WRITE, then by 1-regularity of P, if there exists a WRITE operation preceding R_i, then R_i returns the value written to P by the latest such WRITE. Otherwise, R_i returns v_0, the initial value of the register. Also, the algorithm is obviously wait-free since in a fair execution, it could never block forever in any statement of the pseudocode. Hence, we proved the following:

Theorem 2. *The algorithm in Figure 2 is an implementation of a* 1*-writer/*m*-reader wait-free regular register from one* 1*-writer/*m*-reader wait-free* 1*-regular registers and* $2m$ 1*-writer/*1*-reader safe registers.*

6 Conclusions

We have presented a simple, efficient, and self-contained construction of a wait-free regular register from Byzantine components. This yields a practical building block for distributed storage applications tolerating Byzantine faults.

References

[1] I. Abraham, G. Chockler, I. Keidar, and D. Malkhi. Byzantine disk paxos. In *Proceedings of the 23rd ACM Symposium on Principles of Distributed Computing (PODC'04)*, 2004. To appear.

[2] Y. Afek, D. Greenberg, M. Merritt, , and G. Taubenfeld. Computing with faulty shared objects. *Journal of the ACM*, 42(6):1231–1274, November 1995.

[3] Y. Afek, M. Merritt, and G. Taubenfeld. Benign failures models for shared memory. In *Proceedings of the 7th International Workshop on Distributed Algorithms*, pages 69–83. Springer Verlag, September 1993. In: *LNCS 725*.

[4] H. Attiya and A. Bar-Or. Sharing memory with semi-byzantine clients and faulty storage servers. In *SRDS*, 2003.

[5] R. Bazzi. Synchronous byzantine quorum systems. *Distributed Computing*, 13(1):45–52, 2000.

[6] G. Chockler and D. Malkhi. Active disk paxos with infinitely many processes. In *Proceedings of the 21st ACM Symposium on Principles of Distributed Computing (PODC'02)*, 2002.

[7] E. Gafni and L. Lamport. Disk paxos. *Distributed Computing*, 16(1):1–20, 2003.

[8] S. Haldar and K. Vidyasankar. Constructing 1-writer multireader multivalued atomic variables from regular variables. *J. ACM*, 42(1):186–203, 1995.

[9] S. Haldar and P. Vitanyi. Bounded concurrent timestamp systems using vector clocks. *J. ACM*, 49(1):101–126, 2002.

[10] P. Jayanti, T. Chandra, , and S. Toueg. Fault-tolerant wait-free shared objects. *Journal of the ACM*, 45(3):451–500, 1998.

[11] S. Lakshmanan, M. Ahamad, and H. Venkateswaran. Responsive security for stored data. In *Proceedings of the International Conference on Distributed Computing Systems (ICDCS)*, 2003.

[12] L. Lamport. On interprocess communication – Part II: Algorithms. *Distributed Computing*, 1(2):86–101, 1986.

[13] E. K. Lee and C. Thekkath. Petal: Distributed virtual disks. In *Proceedings of the 7th International Conference on Architectural Support for Programming Languages and Operating Systems (ASPLOS-VII)*, pages 84–92, 1996.

[14] S. Lin, Q. Lian, M. Chen, and Z. Zhang. A practical distributed mutual exclusion protocol in dynamic peer-to-peer systems. In *3rd International Workshop on Peer-to-Peer Systems (IPTPS'04)*, 2004.

[15] N. A. Lynch and M. Tuttle. An introduction to Input/Output Automata. *CWI Quarterly*, 2(3):219–246, 1989.

[16] D. Malkhi and M. Reiter. Byzantine quorum systems. *Distributed Computing*, 11(4):203–213, 1998.

[17] D. Malkhi and M. Reiter. An architecture for survivable coordination in large distributed systems. *IEEE Transactions on Knowledge and Data Engineering*, 12(2):187–202, 2000.

[18] J.-P. Martin, L. Alvisi, and M. Dahlin. Minimal byzantine storage. In *Proceedings of the 16th International Symposium on Distributed Computing (DISC)*, October 2002.

[19] G. L. Peterson. Concurrent reading while writing. *ACM Transactions on Programming Languages and Systems*, 5(1):46–55, 1983.

[20] E. Pierce and L. Alvisi. A framework for semantic reasoning about byzantine quorum systems. In *Brief announcement in Twentieth ACM Symposium on Principles of Distributed Computing (PODC 2001)*, 2001.

[21] R. Rodrigues and B. Liskov. Rosebud: A Scalable Byzantine-Fault-Tolerant Storage Architecture. Technical Report MIT-LCS-TR-932, MIT Laboratory for Computer Science, 2004.

[22] C. Shao, E. Pierce, and J. Welch. Multi-writer consistency conditions for shared memory objects. In *Proceedings of the 17th International Symposium on Distributed Computing (DISC'2003)*. Springer-Verlag, 2003.

[23] C. Thekkath, T. Mann, and E. K. Lee. Frangipani: A scalable distributed file system. In *proceedings of the 16th ACM Symposium on Operating Systems Principles*, pages 224–237, 1997.

[24] J. Tromp. How to construct an atomic variable. In *LNCS 392, Proc. 3rd International Workshop On Distributed Algorithms*, pages 292–302. Springer-Verlag, 1989.

[25] K. Vidyasankar. Concurrent reading while writing revisited. *Distributed Computing*, 4:81–85, 1990.

[26] K. Vidyasankar. Weak atomicity: Ahelpful notion in the construction of atomic shared variables. *SADHANA: J. Eng. Sci. IAS 21*, pages 245–259, 1996.

[27] P. Vitanyi and B. Awerbuch. Atomic shared register access by asynchronous hardware. In *27th IEEE Annual Symposium on Foundations of Computer Science*, pages 233–243, 1986.

[28] L. Zhou, F. B. Schneider, and R. van Renesse. Coca: A secure distributed on-line certification authority. *ACM Transactions on Computer Systems*, 20(4):329–368, 2002.

A Wait-Free Safe Register Construction

In this section we present a wait-free SWMR safe register construction from $n > 4t$ regular registers up to t of which can be Byzantine faulty (see Figure 4). The implementation is based on techniques similar to those of [16].

We now prove that the algorithm in Figure 4 is a safe register construction:

Lemma 5 (Safety). *All traces of the algorithm in Figure 4 are safe.*

Proof. Let R be a READ operation that returns, and $W = \text{WRITE}(v)$ be a WRITE operation that returns before R is invoked, and assume that for no W' operation that follows W, W' is invoked before R returns. We prove that R returns v, thereby upholding safety. Indeed, by the WRITE code, *write*$(\langle ts, v\rangle)$ terminates on at least $n - t > 3t + 1$ base registers before R is invoked. Since READ awaits responses from at least $n - t$ registers, by regularity of x_i, there are at least $t + 1$ correct registers that respond with $\langle ts, v\rangle$ to the base object reads issued during R. Therefore, upon completion of line 4, $\mathsf{safe}(\langle ts, v\rangle)$ is true. Thus, $\langle ts, v\rangle \in C$ after line 5. Finally, since no WRITE operation that follows W is invoked before R returns, there might be at most t (faulty) base registers that return $\langle ts', v'\rangle$ with $ts' > ts$ or $ts' = ts \wedge v' \neq v$. Hence, $\langle ts, v\rangle$ is the only highest timestamped value in C which is safe. By line 8, v must be the R's return value in this case. □

Finally, the construction is trivially wait-free since neither WRITE nor READ are ever awaiting more than $n - t$ responses and at least $n - t$ registers are correct. Hence, we proved the following:

Theorem 3. *The algorithm in Figure 4 is an implementation of a SWMR safe register from $n > 4t$ base regular register up to t of which can be Byzantine faulty.*

Shared objects

SWMR regular registers $x_i \in TSVals$, $1 \le i \le n$; initially $x_i = \langle ts_0, v_0 \rangle$;

WRITE Emulation

Local variables:
$enabled[1 \ldots n],\ pending[1 \ldots n] \in$ Boolean
initially $\forall i\ enabled[i] = pending[i] = false$;
$w \in TSVals$; $ts \in TS$;

WRITE(v):
choose $ts \in TS$ larger than previously used;
$w \leftarrow \langle ts, v \rangle$;
for $1 \le i \le n,\ enabled[i] \leftarrow true$;
repeat
CHECK;
until $|\{i : \neg enabled[i] \wedge \neg pending[i]\}| \ge n - t$;
return ack;

CHECK:
if $(\exists i : enabled[i] \wedge \neg pending[i])$ **then**
$\langle enabled[i], pending[i] \rangle \leftarrow \langle false, true \rangle$;
INVOKE $write(x_i, w)$;
if $(\exists i : x_i$ RESPONDED$)$ **then**
$pending[i] \leftarrow false$;

READ Emulation

Local variables:
$enabled[1 \ldots n],\ pending[1 \ldots n],\ old[1 \ldots n] \in$ Boolean
initially $\forall i\ enabled[i] = pending[i] = false$;
$w[1 \ldots n], tmp[1 \ldots n] \in TSVals$;

Predicate and macro definitions:
$\mathsf{safe}(c) \triangleq |\{i : w[i] = c\}| \ge t + 1$;

READ:
for $1 \le i \le n$, **if**$(pending[i])$ **then** $old[i] \leftarrow true$;
for $1 \le i \le n,\ enabled[i] \leftarrow true$;
for $1 \le i \le n$: $w[i] \leftarrow \perp$;
repeat
CHECK;
until $|\{i : \neg enabled[i] \wedge \neg pending[i]\}| \ge n - t$;
$C \leftarrow \{c \in TSVals : \mathsf{safe}(c, w)\}$;
if $(C \ne \emptyset)$ **then**
return $c.val$: $c.ts = max\{c'.ts : c' \in C\}$;
return v_0;

CHECK:
if $(\exists i : enabled[i] \wedge \neg pending[i])$ **then**
$\langle enabled[i], pending[i] \rangle \leftarrow \langle false, true \rangle$;
INVOKE $tmp[i] \leftarrow read(x_i)$;
if $(\exists i : x_i$ RESPONDED$)$ **then**
if $(\neg old[i])$ **then**
$w[i] \leftarrow tmp[i]$;
$pending[i] \leftarrow false$; $old[i] \leftarrow false$;

Figure 4: The SWMR safe register emulation.